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NORMALITY VERSUS COUNTABLE PARACOMPACTNESS IN PERFECT SPACES

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Introduction. The purpose of this announcement is to present, in a unified fashion, solutions to long outstanding questions concerning the relationship between countable paracompactness and normality conditions in perfect spaces. Each section of this paper is the contribution of a single author and is so designated.

It was established in 1951 by Dowker [4] that in perfect spaces (i.e. spaces in which closed sets are G_δ-sets), normality implies countable paracompactness. However, the validity of the converse has remained an open question until the present. In particular, the relationships between normality, countable paracompactness, and pseudo-normality in Moore spaces has been of considerable interest ([8], [10], [11], [17], [19], and [20], for example). In this paper, the authors (1) produce an example of a countably paracompact, perfect, nonnormal T_3-space, (2) produce an example of a pseudo-normal, separable, noncountably paracompact Moore space, and (3) show the consistency and independence of the existence of a countably paracompact, separable, nonnormal Moore space. In addition, several corollaries are given which answer open questions concerning the hereditary and mapping properties of countable paracompactness in perfect spaces.

I. **(Wage [17]).** The construction given below associates a regular, nonnormal T_2-space X^* to each normal, noncollectionwise normal space X.

THE MACHINE. Suppose X is a normal T_2-space and $\{H^\alpha\}_{\alpha<\lambda}$ is a discrete collection of closed sets which cannot be separated by open sets. Let $D = X\backslash H$, where $H = \bigcup\{H^\alpha: \alpha < \lambda\}$. Denote

$$X^* = (X \times \{0, 1\}) \cup (D \times \{(\alpha, \beta): \alpha, \beta < \lambda \text{ and } \alpha \neq \beta\}).$$

For each $A \subset X$ and $\delta \in \{0, 1\} \cup \{(\alpha, \beta): \alpha, \beta < \lambda \text{ and } \alpha \neq \beta\}$, let A_δ denote $(A \times \{\delta\}) \cap X^*$. Now, define a base $\mathcal{B}$ for the desired topology on X^* as follows

(1) if $x \in X^*\backslash(H_0 \cup H_1)$, let $\{x\} \in \mathcal{B}$; and

(2) if U is an open set in X and $\alpha < \lambda$ such that $U \subset (H^\alpha \cup D)$, let

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