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THE RIESZ DECOMPOSITION FOR VECTOR-VALUED AMARTS

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Let $(\Omega, \mathcal{F}, P)$ be a probability space, $\mathbf{N} = \{1, 2, \ldots\}$, and let $(\mathcal{F}_n)_{n \in \mathbf{N}}$ be an increasing sequence of σ-algebras contained in $\mathcal{F}$. A *stopping time* is a mapping $\tau: \Omega \longrightarrow \mathbf{N} \cup \{\infty\}$, such that $\{\tau = n\} \in \mathcal{F}_n$ for all $n \in \mathbf{N}$. The collection of bounded stopping times is denoted by T; under the natural ordering T is a directed set 'filtering to the right'.

Let $\mathbf{E}$ be a Banach space and consider a sequence $(X_n)_{n \in \mathbf{N}}$ of $\mathbf{E}$-valued random variables *adapted to* $(\mathcal{F}_n)$, i.e., such that $X_n: \Omega \longrightarrow \mathbf{E}$ is $\mathcal{F}_n$-strongly measurable. EX (expectation of X) is the Pettis integral of X; $E_A X$ denotes $E(1_A \cdot X)$. The sequence (X_n) is called an *amart* iff each X_n is Pettis integrable and $\lim_T E(X_\tau)$ exists in the strong topology of $\mathbf{E}$.

The *real* Riesz decomposition theorem for amarts [4] asserts that an amart X_n can be uniquely written as a sum of a martingale Y_n, and an amart Z_n that converges to zero in nearly all possible ways: $Z_n \longrightarrow 0$ a.e. and in L^1, and $Z_\tau \longrightarrow 0$ in L^1.

As a consequence of this result, and of the real amart convergence theorem [1]—the first important result involving discrete parameter amarts—we obtain

THEOREM 1. *Let* $\mathbf{E} = \mathbf{R}$. *If* $(X_n, \mathcal{F}_n)$ *is an amart, then* (*and only then*) *for each increasing sequence* $\tau_n \geqslant n$ *in* T, $E^{\mathcal{F}_n} X_{\tau_n} - X_n \longrightarrow 0$ *a.e. and in* L^1.

The Banach-valued Riesz decomposition is the main result of the present note. The *Pettis norm* of a random variable X is $\|X\| = \sup E|f(X)|$ where the supremum is over all $f \in \mathbf{E}'$ with $|f| \leqslant 1$ [6].

A *potential* is an amart that converges to zero in the Pettis norm. A sequence of adapted random variables is said to be *of class* (B) iff $\sup_T E|X_\tau| < \infty$. We prove

THEOREM 2 (RIESZ DECOMPOSITION). *Let* $\mathbf{E}$ *be a Banach space with the Radon-Nikodym property and let* $(X_n, \mathcal{F}_n)$ *be an* $\mathbf{E}$-*valued amart such that*

$$\liminf E|X_n| < \infty. \tag{1}$$

(i) X_n *can be uniquely written as the sum of a martingale* Y_n *and a*

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