BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 82, Number 4, July 1976

A LANGUAGE FOR TOPOLOGICAL STRUCTURES WHICH SATISFIES A LINDSTRÖM-THEOREM

BY M. ZIEGLER

Communicated January 26, 1976

I. **The language L^t.** Let L_2 be the 2-sorted first order language appropriate for structures $(\mathfrak{A}, \alpha, \epsilon)$, where $\mathfrak{A}$ is a L-structure and α is a set of subsets of A. We call $(\mathfrak{A}, \alpha)$ topological if α is a topology. We call a formula of L_2 topological if it is built up using the set quantifier $\exists X$ only in the form $\exists X(t \in X \wedge \phi)$, X does not occur positively in ϕ. (X occurs positively in ϕ if a free occurrence of X in ϕ is inside the scope of an even number of negation symbols. *Note.* Primitive symbols are $\wedge, \neg, \exists x, \exists X$.) L^t is defined as the set of topological sentences of L_2.[1]

LEMMA. (a) *Define* $\tilde{\beta} = \{\bigcup s | s \subset \beta\}$. *Then for all* $\phi \in L^t$, $(\mathfrak{A}, \beta) \models \phi$ *iff* $(\mathfrak{A}, \tilde{\beta}) \models \phi$. (*I.e.* ϕ *is invariant in the sense of Garavaglia* [1].)

(b) $\tilde{\beta}$ *is a topology iff* $(\mathfrak{A}, \beta) \models \text{top}$, *where* top *is the* L^t-*sentence*

$$\forall x(\exists X \wedge x \in X) \wedge \forall x \forall X(x \in X \longrightarrow \forall Y(x \in Y \longrightarrow \exists Z(x \in Z \wedge \forall y(y \in Z \longrightarrow y \in X \wedge y \in Y)))).$$

In the sequel "model" means "topological model".

COROLLARY (see [1]). (a) $T \subset L^t$ *has a model iff* $T \cup \{\text{top}\}$ *is consistent* (*in the* 2-*sorted predicate calculus of* L_2).

(b) *The set of* L^t-*sentences true in all models is r.e.*

(c) L^t *satisfies the compactness theorem*: T *has a model iff every finite subset of* T *has a model.*

(d) L^t *satisfies the downward Löwenheim-Skolem theorem* (L *countable*): *If* T *has an infinite model, it has a "countable" model* $(\mathfrak{A}, \alpha)$, *i.e.* A *countable*, α *having a countable base.*

By the methods of the next section we can prove a Lindström-

THEOREM. *Let* L^* *be a language for topological structures extending* L^t *and satisfying the compactness theorem and the downward Löwenheim-Skolem theorem. Then* $L^* = L^t$.

AMS (MOS) subject classifications (1970). Primary 02B99, 02H99; Secondary 02G05, 54J05, 54D10.

[1] I have seen, that L^t was first considered for topological spaces by T. A. McKee in two articles in the Z. Math. Logik Grundlagen Math. **21** (1975), 405-408 and ibid. (1976).