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RESULTANTS OF MATRIX POLYNOMIALS

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The $(n + m) \times (n + m)$ matrix

$$R(a, b) = \begin{bmatrix} a_0 & a_1 & \cdots & a_n & & & \\ & a_0 & a_1 & \cdots & a_n & & \\ & & \ddots & \ddots & & \ddots & \\ & & & a_0 & a_1 & \cdots & a_n \\ b_0 & b_1 & \cdots & b_m & & & \\ & b_0 & b_1 & \cdots & b_m & & \\ & & \ddots & \ddots & & \ddots & \\ & & & b_0 & b_1 & \cdots & b_m \end{bmatrix}$$

is called the *resultant matrix* of the two polynomials $a(\lambda) = a_0 + a_1\lambda + \cdots + a_n\lambda^n$ and $b(\lambda) = b_0 + b_1\lambda + \cdots + b_m\lambda^m$ ($a_j, b_j, \in \mathbf{C}^1$, $a_n \neq 0$, $b_m \neq 0$). The determinant of this matrix is called the *resultant* of the polynomials $a(\lambda)$ and $b(\lambda)$. The following classical theorem on resultants is well known: *The number of common roots* (*counting multiplicities*) *of the polynomials* $a(\lambda)$ *and* $b(\lambda)$ *is equal to* dim Ker $R(a, b)$.

This statement does not admit a straightforward generalization to matrix polynomials [1], if the same definition of the resultant matrix $R(a, b)$ is used as in the one-dimensional case. For example the matrix

$$R\left(\begin{pmatrix} \lambda - 1 & 0 \\ 1 & \lambda - 1 \end{pmatrix}, \begin{pmatrix} \lambda & 1 \\ 0 & \lambda - 2 \end{pmatrix}\right)$$

is not invertible although the polynomial matrices do not have common eigenvalues, and the matrix

$$R\left(\begin{pmatrix} \lambda + 1 & 0 \\ 1 & \lambda \end{pmatrix}, \begin{pmatrix} \lambda & -1 \\ 0 & \lambda + 1 \end{pmatrix}\right)$$

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