BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 82, Number 4, July 1976

FORM OF SOLUTIONS TO THE p-ADIC EQUATION $y' = 0$

BY GERALD S. STOLLER

Communicated by John W. Wrench, October 13, 1975

The solutions of $y' = 0$ over the real and/or complex numbers have long been known to be the constant functions. Here we shall present a form for any function, z, that maps a suitable subset of $\mathbf{Q}_p$, the complete field of p-adic numbers, into $\mathbf{Q}_p$ (where p is a positive prime) and is differentiable with derivative zero everywhere. We shall also discuss the image-set of such a function.

This problem has been of interest ever since J. Dieudonné gave an example [1, p. 90], [2, p. 19], [4, pp. 35, 39] of a function, z_0, that maps $\mathbf{Z}_p$ homeomorphically onto its image-set and yet has a zero derivative everywhere. M. van der Put has studied integration of p-adic valued functions [3] using the set of solutions to $y' = 0$ without determining this set [2, p. 20].

Let $\mathbf{N}$ be the set of nonnegative integers, let $\mathbf{R}$ be the set of real numbers, and let $\mathbf{R}_{>b}$ be the set of all real numbers greater than the real number b. Let $C = \{0, 1, 2, 3, \ldots, p-1\}$ and let $\mathbf{Z}_p$ be the set of p-adic integers. Every p-adic integer has a canonical form $\Sigma\{a_j p^j : j \in \mathbf{N}\}$, where each a_j is an element of C. z_0, the function of Dieudonné, is given by

$$z_0\left(\sum\{a_j p^j : j \in \mathbf{N}\}\right) = \sum\{a_j p^{2j} : j \in \mathbf{N}\}.$$

First we let f be a function mapping a subspace of $\mathbf{Z}_p$ into $\mathbf{Z}_p$. It is easily shown that f is uniformly continuous (on its domain) iff

$$\begin{gathered}(\exists l : \mathbf{N} \longrightarrow \mathbf{N})\,(\forall n \in \mathbf{N})\,(\exists g_n : C^{l(n)} \longrightarrow \mathbf{Z}_p)\quad(\forall a \in C^{\mathbf{N}})\\ \sum\{a_j p^j : j \in \mathbf{N}\} \in \mathrm{Dom}(f) \Rightarrow\\ f\left(\sum\{a_j p^j : j \in \mathbf{N}\}\right) = \sum\{g_n(a_0, a_1, a_2, a_3, \ldots, a_{l(n)-1})p^n : n \in \mathbf{N}\}.\end{gathered}$$

Now the concept of uniform differentiability is introduced; it bears the same relationship to differentiability that uniform continuity has to continuity. Formally, f is uniformly differentiable on D' iff D' is a subset of $\mathrm{Dom}(f)$ that contains no isolated points of $\mathrm{Dom}(f)$ and

---

AMS (MOS) subject classifications (1970). Primary 34G05, 12B99; Secondary 28–00, 43A75, 43A85.

Key words and phrases. Solutions of $y' = 0$, p-adic solutions of $y' = 0$, kernel of differentiation operator, Egoroff's theorem, uniform continuity, uniform differentiability.