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Methods of numerical integration, by Philip J. Davis and Philip Rabinowitz, Academic Press, New York, 1975, 459 + xii pp., $34.50.

The study of numerical integration dates from antiquity right up to the present. It is an important topic in numerical analysis and scientific computing to which many mathematicians, scientists, and engineers have contributed.

Why numerical integration? Well, many integrals that arise in the real world simply cannot be evaluated analytically. And, of those integrals which can be analytically evaluated, the analytic "answer" may not be useful for computing. (An example: p. 2 of Davis and Rabinowitz.)

For functions of one variable, numerical integration is called "quadrature", from the Greek quadratos, meaning the square whose area equals the area under a given (positive) curve. For functions of more than one variable, numerical integration is called "cubature". Much more is known about quadratures, whereas cubatures are considerably more important to users, a standard state of affairs in mathematical subjects.

Numerical integration derives some of its appeal from the different levels of abstraction from which it can be approached. For example, functional analysis has been used to obtain error bounds. From classical real analysis, the beautiful theory of orthogonal polynomials leads to the powerful Gauss quadratures. On another hand, computing "rules of thumb" lead to the recent adaptive quadratures.

This reviewer feels that most functions of one variable can be adequately numerically integrated, interpolated, etc., but that many functions of more than one variable cannot, especially if numerical data instead of functions are involved. In the latter case, only data in very special geometric configurations can be handled, e.g., tensor or cross product data. Randomly placed data are treated by a Monte Carlo method, if at all.

On the positive side, what ideas from quadratures have been adapted to cubatures?

1. "Product" regions, such as cubes, cones, and cylinders: Cubatures for them can be built up from quadratures.

2. "Gauss cubatures": For the problem

$$\int_a^b f(x)\,dx \simeq \sum_{k=1}^{n} A_k f(x_k),$$

if the $2n$ unknowns consisting of the A_k and x_k are so chosen that the formula is exact whenever $f(x)$ is a polynomial of degree less than or equal to $2n - 1$, the integration scheme is called a Gauss quadrature. That is, a Gauss quadrature integrates exactly as many of the first monomials as there are parameters to be determined. The nodes (the x_k) of Gauss quadratures are the zeros of the corresponding orthogonal polynomials. Now, for functions of two variables and the problem

$$\iint_R f(x,y)\,dx\,dy \simeq \sum_{k=1}^{n} A_k f(x_k, y_k),$$