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BOOK REVIEWS

Topological vector spaces, by A. Grothendieck, Gordon and Breach, New York, London, Paris, 1973, x + 245 pp., $19.50. (Translated by Orlando Chaljub)

This book is a translation of the notes of a course given by Grothendieck in São Paulo in 1954 and published in French during the same year (a second edition came out in 1958, a third in 1964). The original lecture notes had several distinctions: they constituted the first expository treatment of locally convex spaces, they contained much material which could not be found anywhere else, and, most importantly, they were written by a man who had become one of the leading mathematicians of the 20th century.

Grothendieck was engaged in research on topological vector spaces between 1950 and 1953. During this period he put his stamp on the theory and proved some of its deepest results. In 1954 he wrote his lectures in complete mastery of the whole field as the top expert on the subject: already for this reason alone the notes deserve to appear as a printed book.

It begins with a Chapter 0 entitled "Topological introduction" and containing preliminary material concerning topics which were less well known in 1954 than they are now: initial and final topologies, precompact sets, topologies in function spaces and equicontinuity. The writing is very concise, and since Chapters I–IV, VIII and IX of Bourbaki's "Topologie générale" are stated to be prerequisites anyway, the reader is advised to study the material in Bourbaki's Chapter X, preferably in the "entirely recast" 1961 edition.

Very little of Chapter 0 is used later on, so that one can start with Chapter 1: "General properties", where in 32 pages one gets an elegant introduction to the basic concepts: topologies compatible with vector structure, subspaces, quotients, products, direct sums, continuous linear maps, bounded sets, $\mathfrak{S}$-topologies (which in the translation became $\mathfrak{G}$-topologies). Locally convex spaces are defined through seminorms, and since convex sets appear only in Chapter 2, their characterization as spaces with a fundamental system of convex neighborhoods is given there. Chapter 1 contains the Banach homomorphism theorem, the closed graph theorem and the Banach-Steinhaus theorem, with their usual corollaries.

The title of Chapter 2 is "The general duality theorems on locally convex spaces". After a discussion of convex sets, the geometric form of the Hahn-Banach theorem is proved in a simple way. There exist now several proofs of the analytic form, one based on Banach's original idea, one on a theorem concerning the extension of positive linear forms. Aumann and Dinges considered a generalization of the latter, in which vector spaces are replaced by commutative monoids, the best result in this direction is due to Scheller [**33**]. There is a relation between extensions of additive maps and "sandwich" theorems which affirm the existence of an additive map between a subadditive and a superadditive one [**12**], [**22**].