BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 82, Number 3, May 1976

MARKOV PROCESSES ON MANIFOLDS OF MAPS

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Communicated by Daniel W. Stroock, January 2, 1976

1. **Introduction.** In this note we describe a construction of a Markov process on a manifold of maps starting from a Gaussian measure on the space of sections of an associated vector bundle. Let S be a compact metric space of finite metric dimension and M a smooth complete finite dimensional Riemannian manifold. Our basic construction gives a family $\{\nu_t\colon t \geqslant 0\}$ of Borel probability measures on the space $C(S \times M, M)$ of continuous functions from $S \times M$ to M with the compact-open topology. The multiplication $(f, g)(s, m) = f(s, g(s, m))$ for $f, g \in C(S \times M, M)$ makes $C(S \times M, M)$ into a topological semigroup with identity. Then $\nu_t * \nu_s = \nu_{t+s}$ for $s, t \geqslant 0$ and the right translates of the ν_t give transition probabilities for a Markov process on $C(S \times M, M)$ with continuous sample paths. The left action of $C(S \times M, M)$ on $C(S, M)$ induces a Markov process on $C(S, M)$ with transition probability $\nu_{t,g}$ = image of ν_t under the action of $C(S \times M, M)$ on $g \in C(S, M)$.

2. **Statement of results.** Let ξ denote the product bundle $S \times TM \longrightarrow S \times M$ and $C(\xi)$ the space of continuous sections of ξ. Given a Gaussian measure μ of mean zero on $C(\xi)$, define

$$Q(s, x, t, y) = \int f(s, x) \otimes f(t, y)\, d\mu(f) \in T_xM \otimes T_yM \quad \text{for all } s, t \in S,\ x, y \in M.$$

Q is a reproducing kernel for the bundle ξ (see Baxendale [1]) and determines μ uniquely. Let $X \in C(\xi)$.

For a closed isometric embedding of M inside some Euclidean space V, let $h(x)$ denote the second fundamental form for $M \subset V$ at $x \in M$. Using the natural inclusion $T_xM \subset V$ and orthogonal projection $V \longrightarrow T_xM$, we think of X, Q and h taking values in V and its various tensor products.

THEOREM 1. *Suppose there exists a closed isometric embedding $M \subset V$ such that* (i) *h is bounded and uniformly Lipschitz with respect to the metric on M induced from V.*

Suppose moreover that there exist a Gaussian measure μ on $C(\xi)$, $X \in C(\xi)$ and $\alpha > 0$, $C > 0$ such that

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AMS (MOS) subject classifications (1970). 58B20, 58D15, 60H10, 60J35.