

## EXISTENCE AND REGULARITY OF MINIMAL SURFACES ON RIEMANNIAN MANIFOLDS

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This is a research announcement the full details of which appear in [PJ1] and [PJ2]. We study the existence and regularity of  $k$  dimensional minimal surfaces on smooth riemannian manifolds.

Suppose  $M$  is a smooth compact riemannian manifold of dimension  $n$ , and  $k \leq n$  is a positive integer. It is a problem of long standing to show that  $M$  supports a regular closed minimal submanifold of dimension  $k$ . Until now the only general existence theorem known which required no additional hypothesis on  $M$  applied when  $k = 1$ . In this case the variational methods of M. Morse imply the existence of closed geodesics on  $M$ , possibly with self-intersections. Results established for  $k \geq 2$  have required additional geometric or topological hypotheses on  $M$ . For example, if  $k = n - 1 \leq 6$ , then the support of every nonzero homologically area minimizing integral cycle is a smooth closed embedded  $k$  dimensional submanifold, a fact established by the methods of [FH, Chapter 5]. Thus one settles the existence problem affirmatively in these dimensions whenever the  $k$  dimensional homology group of  $M$  with coefficients in the integers does not vanish. Another example is the theorem of Lawson [LHB] that  $M = S^3$  supports closed two dimensional minimal submanifolds of arbitrarily high genus.

There are few results known in the general case, where  $M$ ,  $k$ , and  $n$  are arbitrary. We announce an application of variational methods similar to those of Almgren [AF] and Morse to show that  $M$  supports a nonzero stationary  $k$  dimensional integral varifold which at each point of  $M$  is *almost minimizing* in all sufficiently small annular neighborhoods of that point. Almost minimizing varifolds are principal objects of our investigation. Intuitively one considers an almost minimizing varifold to be one which may be approximated arbitrarily closely by integral currents, which are themselves very nearly locally minimizing. A varifold which is only stationary and integral has in general essential singularities, possibly of positive measure. If in addition the varifold is almost minimizing, then it possesses strong local stability properties which yield estimates on the

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