

ON SCARCITY OF OPERATORS WITH FINITE SPECTRUM

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Let ρ denote the spectral radius of an operator; in 1968–1970 Edoardo Vesentini proved

PROPOSITION 1 ([5] AND [6]). *If $\lambda \rightarrow f(\lambda)$ is an analytic function mapping a domain in \mathbb{C} into a complex Banach algebra then $\lambda \rightarrow \text{Log } \rho(f(\lambda))$ is subharmonic.*

From this we got the following generalization of Newburgh's continuity theorem [4], where $\sigma(x)$ is the union of $\text{Sp } x$ and its holes.

PROPOSITION 2 (ALMOST-CONTINUITY THEOREM). *If $\lambda \rightarrow f(\lambda)$ is analytic on a domain D containing z_0 and if E is a subset of D , such that $z_0 \in \bar{E}$, E is nonsharp at z_0 , then there exists a sequence (λ_n) converging to z_0 with $\lambda_n \in E$, $\lambda_n \neq z_0$, and $\lim_{n \rightarrow \infty} \sigma(f(\lambda_n)) = \sigma(f(\lambda_0))$.*

The same statement with the spectrum is false. If δ is the diameter of the spectrum, we obtained as well

PROPOSITION 3. *With the same hypothesis $\lambda \rightarrow \text{Log } \delta(f(\lambda))$ is subharmonic.*

All of these results and intricate properties of subharmonic functions, capacity and sharp sets, easily found in [1], give the fundamental theorem

THEOREM 1. *Either the set of λ , such that the spectrum of $f(\lambda)$ is finite, is of outer capacity zero, or there exists an integer n such that the spectrum of $f(\lambda)$ has exactly n elements, for every λ , except on a closed set of capacity zero, where the spectrum has at most $n - 1$ elements.*

Kaplansky [3], in 1954, and Hirschfeld-Johnson [2], in 1972, proved that $A/\text{Rad } A$ is finite dimensional, for a complex Banach algebra A , if the spectrum of every element of this algebra is finite. Unfortunately the method does not work for local and real cases. Other persons (Behncke, Wong) obtained the same result for A^* -algebras supposing the spectrum finite for hermitian elements.

Theorem 1 can be used to get

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