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PROBABILISTIC FOUNDATIONS OF QUANTUM THEORIES AND RUBIN-STONE SPACES

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Abstract. A construction of Mielnik probability spaces of dimension 2 is given in terms of Rubin-Stone functionals, and conversely it is shown that if a certain subset of a real linear space is supplied with 2 dimensional Mielnik probability space structure then the space becomes a Rubin-Stone space (in final analysis, a generalized inner product space). Analogous results are given for generalized inner product spaces in the sense of Nagumo.

1. **Introduction.** In [1], H. Rubin and M. H. Stone define a class of generalized inner product spaces. In this note, we announce some results concerning the use of these spaces as concrete representation spaces for quantum states. Our approach is a probabilistic one and follows the ideas and methods in [2] and [3].

DEFINITION 1.1. A linear space N over the real, complex, or quaternionic number system is a *Rubin-Stone space* if the following four postulates are satisfied:

POSTULATE 1. On N there is defined a nonnegative real function q such that

$$q(x+y)+q(x-y)=2q(x)+2q(y).$$

POSTULATE 2. As a function of the real number α the quantity $q(\alpha x)$ is bounded in some neighborhood of $\alpha = 0$ for each x.

POSTULATE 3. In the complex and quaternionic cases the relations $q(x) = q(ix)$, $q(x) = q(ix) = q(jx) = q(kx)$ hold for the imaginary units i and i, j, k respectively.

POSTULATE 4. If $q(x) = 0$, then $x = \theta$.

DEFINITION 1.2. Let S be a nonempty set, and let p be a real-valued function defined on $S \times S$ such that

(A) $0 \leqslant p(a, b) \leqslant 1$ and $a = b \Longleftrightarrow p(a, b) = 1$,

(B) $p(a, b) = p(b, a)$ for all $a, b \in S$.

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[1] Portions of these results will appear in a doctoral thesis of S. J. Guccione, Jr. at the University of Missouri-Rolla.