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BOOK REVIEWS

Topics in operator theory, by C. Pearcy, Mathematical Surveys, No. 13, Amer. Math. Soc., Providence, Rhode Island, 1974, 235+ix pp., $23.00.

Topics in operator theory consists of five diverse expository papers surveying a number of recent and not so recent ideas in the theory of bounded linear operators on Hilbert space. The essays range from an exposition of multiplicity theory for normal operators by Arlen Brown to an inclusive and detailed review of the current state of our knowledge of weighted shift operators by Allen L. Shields. Also one finds an illuminating introduction to invariant subspaces and their relation to various problems in analysis by Donald Sarason, a concise geometric introduction to the model theory of Sz.-Nagy and Foiaş by R. G. Douglas and finally a report on the recent powerful technique of V. I. Lomonosov for producing invariant subspaces of operators related to compact operators by Carl Pearcy and Allen L. Shields.

Two results concerning operators on finite dimensional complex vector spaces lead one to feel that one has a fairly good grasp of their structure. First, every operator can be represented by an upper triangular matrix with respect to some orthonormal basis; and second, if arbitrary bases are admitted, then every operator can be represented by a matrix in Jordan normal form.

One of the principal goals of operator theory is to obtain a comparable understanding of the structure of operators on arbitrary Hilbert or Banach spaces. The invariant subspace problem is an indication of just how far we are from attaining such a goal. For a trivial consequence of either of the two results on finite dimensional spaces is that every operator on a space $\mathcal{H}$ of dimension at least two has a nontrivial invariant subspace, a subspace other than (0) or $\mathcal{H}$ that is mapped into itself. Whether or not every operator on an infinite dimensional space has a closed invariant subspace remains an open question despite intense effort in the past few years.

With the general problem so intractable the focus naturally shifts to special classes of operators. In the book under review we find an examination of several of these, some illustrations of how they relate to other parts of analysis, and an introduction to one attempt at a general model theory, which the special cases have led to. The preparation required of the reader varies a bit between the articles, but generally requires the standard courses on measure theory, complex function theory and functional analysis. As the audience will probably differ somewhat from essay to essay, we will comment on this as we take up each in order of appearance. In general the book provides a good introduction to some (but by no means all) of the topics which have interested operator theorists in the past few years.

The first article, *Invariant subspaces*, by Donald Sarason, deals mainly with operators tied to the unit circle of the complex plane and related