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ON THE TAMAGAWA NUMBER OF QUASI-SPLIT GROUPS

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1. **Introduction.** In this paper we give a formula for the Tamagawa number $\tau(G)$ (see [6]) of a connected semisimple quasi-split algebraic group G defined over an algebraic number field F. The method used is that of R. P. Langlands (see [2]).

Let $\mathbf{A}$ be the adeles of F; $G_{\mathbf{A}}$ the locally compact adele group of G in which the group G_F of F-rational points is embedded.

Let B be the Borel subgroup of G defined over F, and A the maximal torus of B defined over F. $\tau(A)$ is the Tamagawa number of A. L_F (resp. L_F^+) denotes the lattice of F-rational weights of G (resp. of the simply-connected form of G). Let c be the index $[L_F^+ : L_F]$. Then the main formula is

THEOREM. $\tau(G) = c\tau(A)$.

2. **Sketch of the proof.** Let P be the orthogonal projection of $L^2(G_F\backslash G_{\mathbf{A}})$ onto the space of constant functions. Langlands [2] observes the simple relation:

$$(1,1)(P\varphi^\sim, P\psi^\sim) = (\varphi^\sim, 1)(1, \psi^\sim) \tag{1}$$

where $\varphi^\sim$, $\psi^\sim \in L^2(G_F\backslash G_{\mathbf{A}})$ and $(\cdot\,,\cdot)$ is the inner product on $L^2(G_F\backslash G_{\mathbf{A}})$. As

$$(1,1) = \int_{G_F\backslash G_{\mathbf{A}}} dg, \tag{2}$$

the problem reduces to the computation of the remaining three terms in (1).

Let $G_\infty = \Pi_{v|\infty} G_{F_v}$ where F_v is the completion of F at the place v and "$v|\infty$" means that v is infinite. Let K_∞ be the maximal compact subgroup of G_∞, and $K_0 = \Pi_{v<\infty} G_{\mathcal{O}_v}$ where "$v < \infty$" means that v is finite, $\mathcal{O}_v$ is the maximal compact subring of F_v and $G_{\mathcal{O}_v}$ is the compact subgroup of G_{F_v} consisting of elements with coefficients in $\mathcal{O}_v$ and whose determinants are units. Put

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[1] This paper is based on the author's Ph. D. dissertation, written at Yale University under Professor G. D. Mostow. The problem and the approach were suggested by R. P. Langlands.