BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 82, Number 2, March 1976

EXTENSION OF HOLOMORPHIC MAPS

BY BERNARD SHIFFMAN[1]

Communicated October 27, 1975

In this note we announce results on extending holomorphic maps into compact complex manifolds that satisfy certain curvature conditions. These results are analogous to theorems on extending holomorphic maps into manifolds with holomorphic sectional curvatures $\leqslant 0$ obtained by the author [5] and independently by Griffiths [3]. A similar type of result on meromorphic extension of equidimensional maps has been given by Griffiths [2, Theorem D].

Let E be a hermitian holomorphic vector bundle on a complex manifold M. Let $R(v, \bar{w}, s, \bar{t})$ denote the curvature tensor for the hermitian connection on E, where $v, w \in E$, $s, t \in T =$ holomorphic tangent bundle of M. (Our sign convention is chosen so that if $E = T$, then $R(s, \bar{s}, s, \bar{s})$ is the usual holomorphic sectional curvature of a unit tangent vector s.) Let $\bigwedge_k T$ denote the bundle of holomorphic k-vectors on M ($\bigwedge_1 T = T$) and suppose $\bigwedge_k T$ is given a hermitian metric. For a simple vector $v \in \bigwedge_k T_x$ ($x \in M$), we let Σ_v denote the k-dimensional subspace of T_x associated with v, and we consider the hermitian form R_v on Σ_v given by

$$R_v(s, \bar{t}) = R(v, \bar{v}, s, \bar{t}) \quad \text{for } s, t \in \Sigma_v.$$

We say that a complex manifold M has property H_k if $\bigwedge_k T$ carries a hermitian metric such that for all simple k-vectors v, R_v has at least one nonpositive eigenvalue. Note that M has property H_1 if and only if M has a hermitian metric with holomorphic sectional curvatures $\leqslant 0$.

THEOREM 1. *Let $1 \leqslant j < k < n$, and let M be a compact complex manifold that has properties H_j and H_k. Let D be a domain with smooth boundary in $\mathbf{C}^n$. Let $p \in \partial D$, and suppose D is strictly $(n-k)$-pseudoconcave at p (i.e., if $D = \{u < 0\}$, then the Levi form of u restricted to the "holomorphic" tangent space of ∂D at p has at least k negative eigenvalues). Then every holomorphic map $f: D \longrightarrow M$ extends holomorphically to a neighborhood of p.*

The analogous result for $k = 1$ is given in [5, Lemma 3] where M is allowed to be complete instead of compact (see also [3]).

AMS (MOS) subject classifications (1970). Primary 32H99, 32D15, 53C55.

[1] Research partially supported by NSF Grant and by a Sloan Fellowship.