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THREE WEIL REPRESENTATIONS ASSOCIATE TO FINITE FIELDS

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ABSTRACT. Following the method of A. Weil [4], we define the Weil representation of general linear groups in 1, of symplectic groups (odd characteristic) in 2, of unitary groups in 3, over finite fields. We give its character and decomposition and some functorial properties. The symplectic case was also studied by R. Howe [1] and M. Saito [3], the unitary case by R. I. Lehrer [2].

1. The Weil representations of symplectic groups (odd characteristic).

1.1. Let (E, j) be a symplectic vector space over the field k with q elements. Let $H(E, j)$ be the group $E \times k$ with the law

$$(w, z)(w', z') = (w + w', z + z' + i(w, w')), \tag{1}$$

where $i = j/2$. It is a two-step nilpotent group with center Z isomorphic to k by $z \longmapsto (0, z)$. The group $\mathrm{Sp}(E, j)$ of j acts on $H(E, j)$ by $s\colon (w, z) \longmapsto (sw, z)$.

1.2. For each nontrivial character ζ of Z, there is a unique class $\eta_\zeta^{(E,j)}$ of irreducible representations of $H(E, j)$ given by ζ on Z.

THEOREM 1. *There is a unique extension* $W_\zeta^{(E,j)}$ *of* $\eta_\zeta^{(E,j)}$ *to* $\mathrm{Sp}(E, j)$, *except for* $q = 3$, $\dim E = 2$, *where there is a unique extension* $W_\zeta^{(E,j)}$ *disjoint from its conjugate.*

The representation $W_\zeta^{(E,j)}$ is called the Weil representation of $\mathrm{Sp}(E, j)$ associated to the character ζ.

1.3. The Weil representations $W_\zeta^{(E,j)}$ have the following properties:

(1) $W_\zeta^{(E,j)} = W_{\zeta'}^{(E,j)}$ iff $\zeta'((0, z)) = \zeta((0, zt^2))$ for a $t \in k^*$ and all $z \in k$.

(2) $W_\zeta^{(E,j)}$ splits in two simple components of degree $(q^n + 1)/2$ and $(q^n - 1)/2$, where $n = \dim E$, given on the center of $\mathrm{Sp}(E, j)$ respectively by $(1/q)^n$ and $-(1/q)^n$.

(3) The complex conjugate of $W_\zeta^{(E,j)}$ is $W_{\zeta^{-1}}^{(E,j)}$.

(4) The support of the character of $W_\zeta^{(E,j)} \otimes \eta_\zeta^{(E,j)}$ is the set of conjugates of $\mathrm{Sp}(E, j)Z$.

(5) The class $W_\zeta^{(E,j)} \otimes W_{\zeta^{-1}}^{(E,j)}$ is the natural representation of $\mathrm{Sp}(E, j)$ in $\mathbf{C}[E]$.

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