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BOOK REVIEWS

Algebraic topology—homotopy and homology, by Robert M. Switzer, Die Grundlehren der math. Wissenschaften, Band 212, Springer-Verlag, Berlin, 1975, xiv+526 pp., \$52.50.

This is a very advanced algebraic topology text. Assuming a reader with at least a year's background in algebraic topology, the author proposes to develop the fundamentals of stable homotopy theory, so that by the end of the book the reader would be ready to do research.

In order to try to describe stable homotopy theory, one may begin with a rather obvious question. Being given two spaces X and Y, one would like to describe the set of maps from X to Y, i.e., the continuous functions $f:X\to Y$. Clearly one has the constant maps ($f(x)=y_0$ for all $x\in X$) and fixing a point $x_0\in X$ one can assign to each map f the point of Y given by $f(x_0)$. An immediate, and essential, simplification is then to fix points $x_0\in X$ and $y_0\in Y$, and ask for a description of the set of maps $f:X\to Y$ with $f(x_0)=y_0$.

The question is far too general for a reasonable solution, and the set of maps is frequently so large as to be unmanageable. Homotopy theory approaches this question by introducing an equivalence relation on maps. Given two maps $f_0, f_1:X\to Y$ ($f_i(x_0)=y_0$), one says f_0 and f_1 are homotopic if there are maps $f_t:X\to Y$, $0\leqq t\leqq 1$, with $f_t(x_0)=y_0$, so that the resulting function $F:X\times[0,1]\to Y:(x,t)\to f_t(x)$ is continuous. The set of equivalence classes, denoted $[X;Y]$ suppressing the chosen points from the notation, is called the set of homotopy classes of maps from X to Y. The hope is that $[X;Y]$ captures much of the structure of the set of maps while at the same time being smaller and more tractible, and this is frequently the case.

If Y is a topological group with y_0 the unit, multiplication of functions makes $[X;Y]$ into a group. One can also define a group structure on $[X;Y]$ if X is suspension, i.e., if there is a space X', $x_0'\in X'$, for which X is the quotient space of $X'\times[0,1]$ obtained by identifying $X'\times\{0\}\cup X'\times\{1\}\cup\{x_0'\}\times[0,1]$ to a point x_0. One writes $X=\Sigma X'$. If X' is also a suspension, then for each Y, $[X;Y]$ can be given the structure of an abelian group, and it could be studied and described as an algebraic object.

Being given a map $f:X\to Y$ ($f(x_0)=y_0$ as always) the function $f\times\text{identity}:X\times[0,1]\to Y\times[0,1]$ induces a map $\Sigma f:\Sigma X\to\Sigma Y$, giving a function $\Sigma:[X;Y]\to[\Sigma X;\Sigma Y]$. One may, of course, iterate this construction, and one lets $\{X;Y\}$ be the limit of the sets $[\Sigma^r X;\Sigma^r Y]$. This is the set of stable homotopy classes of (stable) maps of X into Y. It is always an abelian group and is quite manageable, although by no means computable. For example, a major open question is to compute these groups when X and Y are spheres, S^{n+k} and S^n respectively.

In attempting to study maps from X to Y one is then led to consider not