BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 82, Number 2, March 1976

COMBINATORIAL INEQUALITIES AND SMOOTHNESS OF FUNCTIONS

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Introduction. This is an expository account of work we have carried out jointly with some of our students and associates, notably E. Rodemich, H. Taylor, C. Preston, C. Greenhall, S. Milne, T. Park and P. DeLand.

We shall be concerned with classes of functions $f(X)$ measurable on the d-dimensional unit cube

$$I_d = [0, 1]\times[0, 1]\times\cdots\times[0, 1]$$

and satisfying conditions of the type

$$I_{\Psi,p}(f) = \int_{I_d}\int_{I_d} \Psi\left(\frac{f(X)-f(Y)}{p(\overline{XY})}\right) dX\, dY \leqq B < \infty,^2$$

where Ψ and p are restricted as follows:

(1) (a) $\Psi(u)$ is defined and continuous on $(-\infty, +\infty)$,
(b) $\Psi(u) = \Psi(-u)\uparrow\infty$ as $|u|\uparrow\infty$,

and

(2) (a) $p(u)$ is defined and continuous on $(-\sqrt{d}, \sqrt{d})$,
(b) $p(u) = p(-u)\downarrow 0$ as $|u|\downarrow 0$.

In most of our work we have been concerned with the one-dimensional case, but here we shall be able to say something about the general case.

Roughly speaking, our aim has been to use the finiteness of $I_{\Psi,p}(f)$ to derive a priori bounds for other important functionals of f, such as its modulus of continuity or other *high order norms.* The bounds we have obtained involve, of course, Ψ and p but they depend on f *only* through the value of $I_{\Psi,p}(f)$.

1. **Sample of results.** To get across at least the flavor of our work we shall give a sample of our results in the one-dimensional case.

THEOREM 1.1. *Let*

$$I_{\Psi,p}(f) = \int_0^1\int_0^1 \Psi\left(\frac{f(x)-f(y)}{p(x-y)}\right) dx\, dy \leqq B < \infty,$$

An address delivered at the 80th Annual Meeting of the American Mathematical Society in San Francisco on January 15, 1974; received January 20, 1975.

AMS (MOS) subject classifications (1970). Primary 26A54, 26A15, 42A16.

[1] Work supported in part by Air Force Grant AF AFOSR 1322-67 and in part by NSF Grant.

[2] $\overline{XY}$ denotes the Euclidean distance from X to Y.

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