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OPEN THREE-DIMENSIONAL MANIFOLDS WITH FINITELY GENERATED FUNDAMENTAL GROUPS

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Recent results of G. P. Scott [5] and T. W. Tucker [6] indicate that a 3-manifold with a finitely generated fundamental group is in various senses close to being compact. The results announced in this paper are further investigations into the relations between these two properties for open 3-manifolds. With a few additional complications these results also hold for noncompact 3-manifolds with boundary.

THEOREM 1. *Suppose M is an open, connected 3-manifold, $\pi_1(M)$ is finitely generated, M contains no infinite collection of disjoint fake 3-cells, and M contains no 2-sided projective planes. Then M is homotopy equivalent to the interior of a compact 3-manifold with a tame, closed, 0-dimensional subset deleted. The compact 3-manifold is homeomorphic to a submanifold of M.*

Theorem 1 is an extension of Theorem 3.2 of [1]. In general, of course, the homotopy equivalence will not be proper: Whitehead's example [7] of a contractible, open 3-manifold shows that even in simple cases, the structure at infinity can be quite complicated.

After finding a maximal, finite collection of disjoint fake 3-cells in M and replacing them with 3-cells, the conclusion of Theorem 1 follows from the results of [4] that are stated as Theorems 2 and 3 below.

DEFINITIONS. Suppose M is an open, connected 3-manifold. A *submanifold* of M is a 3-manifold embedded as a polyhedral subset of M. A *punctured collar* on the boundary of a submanifold S of M is a submanifold C in $M - \text{Int}(S)$ such that $C \cap S = \partial S$ and $(C, \partial S)$ is homeomorphic to $(\partial S \times I - \text{Int}(B^3), \partial S \times \{0\})$ where B^3 is a union of disjoint 3-cells in $\text{Int}(\partial S \times I)$. A *nucleus* of M is a compact submanifold S such that any compact subset of M can be engulfed by adding a punctured collar to ∂S and then attaching 1-handles to the boundary of the resulting submanifold.

THEOREM 2. *Suppose M is an open, connected 3-manifold, $\pi_1(M)$ is finitely generated, and M contains no fake 3-cells and no 2-sided projective planes. Then M has a nucleus.*

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