## UNIQUE FACTORIZATION IN RANDOM VARIABLES

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The problem of determining the potential q(x) from "spectral data" for the equation

$$-y''(x) + q(x)y(x) = \lambda y(x), \quad -\infty < x < \infty,$$

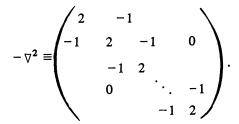
has been studied extensively. For a review, see [1] and [5].

A typical kind of result, associated with the names of Gelfand and Levitan tells you that if the discrete spectrum, the normalizing constants, and the reflection coefficient R(k) are known, then q(x) can, in principle, be determined uniquely.

Here we consider a random version of this problem. We envisage a situation where one keeps records of "spectral data" for noisy versions of the potential q(x) and attempts to determine the "mean potential" from the distribution of the data.

It turns out that in this random case a smaller set of quantitites—measured over and over again—give a lot of information about q(x). A similar situation develops in a variety of different setups (see, for instance, [3] and [4]).

Let  $-\nabla^2$  stand for the  $n \times n$  matrix



THEOREM I. Let  $q_1, \ldots, q_n$  be independent Gaussian random variables with unknown means  $\overline{q}_1, \ldots, \overline{q}_n$  and variances all equal to one. Then the joint distribution function of  $tr(-\nabla^2 + qI)^k$ ,  $k = 1, \ldots, n$ , determines the vector  $\overline{q}_1, \ldots, \overline{q}_n$  up to a global reflection  $\overline{q}'_i \equiv \overline{q}_{n-i+1}$ .

The theorem above says that the spectrum determines the potential up to a trivial reflection. This is to be compared with the nonrandom case where, in

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