BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 81, Number 5, September 1975

A STRUCTURE SHEAF FOR A NONCOMMUTATIVE NOETHERIAN RING

BY BETH GOLDSTON AND A. C. MEWBORN

Communicated by Barbara L. Osofsky, June 2, 1975

Throughout we assume that R is a left noetherian ring, not necessarily commutative. R-modules are left modules, $[M]$ denotes the isomorphism class and $E(M)$ the injective hull of a module M.

The (left) *spectrum* of R, denoted Spec R, is taken to be the set of isomorphism classes of indecomposable injective modules. We denote by $G(R)$ the directed graph whose set of vertices is the set Spec R and such that if $[V_1]$, $[V_2] \in \operatorname{Spec} R$ there is a directed edge from $[V_1]$ to $[V_2]$ in $G(R)$ if and only if there exist critical (see [2, p. 9]) submodules S_1 and S_2 of V_1 and V_2 and a short exact sequence $0 \longrightarrow S_1 \longrightarrow A \longrightarrow S_2 \longrightarrow 0, A \subseteq V_1$. For a left ideal I of R define $K(I)$ to be the subset of Spec R consisting of $[V]$ such that there exists $[W] \in \operatorname{Spec} R$, a nonzero map $\alpha: R/I \longrightarrow W$, and a directed path in $G(R)$ from $[W]$ to $[V]$. If there is a nonzero module map from W to V there is a directed path in $G(R)$ from $[W]$ to $[V]$.

PROPOSITION 1. *The collection of subsets of* Spec R *of the form $K(I)$, I a left ideal of R, is a basis for the closed sets of a topology on R. If R is commutative then* Spec R *is homeomorphic to the classical spectrum with its Zariski, or hull-kernel, topology.*

For each $x \in \operatorname{Spec} R$ we fix an indecomposable injective module V_x such that $[V_x] = x$. For a subset Y of Spec R we define $V_Y = \coprod_{y \in Y} V_y$, $E_Y = \operatorname{End}_R(V_Y)$, and $R_Y = \operatorname{End}_{E_Y}(V_Y) = \operatorname{Biend}_R(V_Y)$. We regard V_Y as a right E_Y-module. We define a presheaf of rings over Spec R by letting, for each open subset U, the ring of sections over U be R_U, with the obvious restriction maps.

THEOREM 2. *The above presheaf is a sheaf and for each open subset U of* Spec R, *R_U is naturally identified with the quotient ring of R with respect to the torsion theory determined by the injective module V_U. In particular, $R_{\operatorname{Spec} R}$ is naturally identified with R.*

If R is commutative the sheaf constructed above reduces to the usual structure sheaf.

If M is an R-module the *support* of M, denoted Supp M, is the set of those $[V] \in \operatorname{Spec} R$ such that M is not V-torsion. (For definitions and basic properties on torsion theories, see [6].) When R is commutative, the above definition of the

AMS (MOS) subject classifications (1970). Primary 16A08, 16A46, 16A52.