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SPRINGER-TYPE THEOREMS FOR SPINOR GENERA OF QUADRATIC FORMS

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At the Quadratic Forms Conference (Baton Rouge, Louisiana, 1972), N. C. Ankeny raised the question on the behaviour of the genus of a positive definite integral quadratic form upon inflation to a totally real number field. Here we announce some results of the closely related problem of how the *spinor* genus behaves when lifted to an overfield. We treat this question geometrically and also in a more general setting; namely, we study the spinor genera associated with an arbitrary quadratic lattice, not necessarily a free lattice. A classical theorem of Springer [S] asserts an anisotropic quadratic space over F remains anisotropic in E if the field degree $[E: F]$ is odd. In terms of classical Witt rings, it says the natural map $W(F) \to W(E)$ is injective. Our results for the spinor genus behaviour are similar in spirit. Detailed proofs will appear elsewhere. Unexplained notations are from [0].

Let E/F be a finite extension of global fields, $\mathcal{O}_E$, $\mathcal{O}_F$ the integers in E, F respectively, L a lattice on a regular space V over F with rank $r(L) \geqslant 3$. Put $\widetilde{V} = V \otimes E$ and $\widetilde{L} = L \otimes \mathcal{O}_E$. Define maps β: $J_F \to J_E$ and γ: $J_V \to J_{\widetilde{V}}$, respectively, by $(\beta(j_p))_P = j_p$ and $(\gamma(u_p))_P = u_p \otimes E_P$ for $P|p$. Then β and γ induce vertical maps ψ_L and Γ_L, respectively, in the commutative diagram

$$\begin{tikzcd} J_V/P_V J'_V J_L \arrow[r, dashed, tail, two heads] \arrow[d, "\Gamma_L"'] & J_F/P_D J_F^L \arrow[d, "\psi_L"] \\ J_{\widetilde{V}}/P_{\widetilde{V}} J'_{\widetilde{V}} J_{\widetilde{L}} \arrow[r, dashed, tail, two heads] & J_E/P_{\widetilde{D}} J_E^{\widetilde{L}} \end{tikzcd}$$

so that Γ_L is injective if and only if ψ_L is injective as the horizontal maps are isomorphisms. Our main results are:

THEOREM A. *Let L be a quadratic lattice of rank $r(L) \geqslant 3$ and defined over a global field F with the property that at each dyadic localization L_p is modular. Then for any odd degree field extension E/F, ψ_L is injective.*

THEOREM B. *Let L be a quadratic lattice of rank $r(L) \geqslant 3$. If E/F is an odd degree field extension of number fields such that 2 is unramified in E, then ψ_L is injective.*

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