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AN EXTENSION OF KHINTCHINE'S INEQUALITY[1]

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Khintchine's inequality [4] states that if $\{X_j: j = 1, \ldots, N\}$ are independent identically distributed Bernoulli random variables ($X_j = \pm 1$ with equal probabilities), then for any choice of real a_j, and any $m = 2, 3, \ldots, X = \Sigma_j a_j X_j$ satisfies

$$E(X^{2m}) \leqslant ((2m)!/2^m m!)(E(X^2))^m. \tag{1}$$

This inequality implies [9, Chapter 5] that for $0 < p < \infty$, there exist positive constants A_p and B_p depending only on p (with $B_{2m} = ((2m)!/2^m m!)^{1/2m}$) such that

$$A_p\|X\|_2 \leqslant \|X\|_p \leqslant B_p\|X\|_2 \tag{2}$$

where $\|X\|_p$ denotes the p-norm, $(E(|X|^p))^{1/p}$. Khintchine's inequality in this form has many applications in which the $\{X_j\}$ are generally represented as Rademacher functions [9], [7], [3].

In this note we give an extension of Khintchine's inequality from the Bernoulli case to that of random variables of the following type:

DEFINITION. A random variable X is of *type* L if its moment generating function $E_X(z) \equiv E(\exp(zX))$ satisfies

(i) $\exists C \ni E_X(z) \leqslant \exp(Cz^2)$ for all real z and

(ii) $E_X(z) = 0 \Rightarrow z = i\alpha$ for some real α.

Symmetric random variables satisfying condition (i) have been called *subgaussian* by Kahane; they satisfy an inequality similar to but weaker than (1) [2, p. 87].

Theorem 1 below extends Khintchine's inequality to arbitrary linear combinations of independent random variables of type L while Theorem 2 treats the case of positive linear combinations of type L random variables with a particular kind of dependence (such as arises in models of ferromagnets). Complete proofs of these theorems together with further results concerning random variables of type L and applications of these results to statistical mechanics and quantum field theory will appear in [6].[2]

THEOREM 1. *If $\{X_j\}_{j=1}^N$ are independent (not necessarily identically distributed) random variables of type L, then the inequality* (1) *applies for any*

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[2]Other related results are contained in a paper, *Gaussian correlation inequalities for ferromagnets*, by the author, which will appear in Z. Wahrscheinlichkeitstheorie und Verw. Gebiete.