studied where the quadratic form involves only the final value of the physical process over a fixed bounded time interval. The solution of this problem is applied to a control problem where the physical system $X(t)$ is affected by the sum of two controls. The control functions are regarded as the player strategies in a two person zero sum game and the stochastic system is described as a differential game with imperfect information.

The book by Arnold may serve as a textbook or reference work. Its substantial bibliography contains reference lists for topics such as Markov and diffusion processes, stochastic differential equations, stability, filtering, control, and probability theory. There is a good index and each section of the book is well organized. The book is especially valuable for nonexperts on stochastic differential equations who wish to deal with models for processes affected by noise. One can learn the limitations of the theory as well as recent results on a variety of problems. The notes of Balakrishnan are valuable to anyone who desires to master the mathematical techniques involved in modern stochastic control theory.

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Optimal control theory, by L. D. Berkovitz, Applied Mathematical Sciences, 12, Springer-Verlag, New York, 1974, ix+304 pp., \$9.50

The term "mathematical theory of optimal control" has come to refer to the optimization of a certain class of functionals of state and control variables for dynamical systems whose evolution with time is described by ordinary differential equations. Such problems are similar to the Bolza problem in classical calculus of variations, with the important difference that inequality constraints may be imposed. A large literature on optimal control theory developed during the 1960's, stimulated by the slightly earlier work of Bellman, Pontryagin, and their associates. Most of the questions with which that literature was concerned have by now been resolved. It is the task of authors of books on control theory to preserve the essential aspects, for those interested in the applications and as a foundation for students entering an area of active current research (e.g. control of systems governed by partial differential equations, control systems with time delays, and stochastic control).

In this book Berkovitz gives a readable account not only of the standard Pontryagin necessary conditions for a minimum but also of the problem of existence. The proof given for the Pontryagin necessary conditions follows Gamkrelidze, SIAM J. Control (1965). Like other proofs, it depends on the idea of convex set of variations (due to McShane in 1939) and the Brouwer fixed point theorem.

The traditional method in calculus of variations for proving existence of a minimum is to show precompactness of minimizing sequences and lower semicontinuity. A nicer technique was found in 1959 by Filippov; it avoids lower semicontinuity but uses a theorem about measurable selections. This