BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 81, Number 5, September 1975

BOOK REVIEWS

Stochastic differential equations: theory and applications, by L. Arnold, Wiley-Interscience, New York, 1974, xvi+228 pp., \$17.95

Stochastic differential systems. I: Filtering and control, a function space approach, by A. V. Balakrishnan, Lecture Notes in Economics and Mathematical Systems, vol. 84, Springer-Verlag, Berlin, 1973, v+252 pp., \$8.20

The description of Brownian motion given by Einstein is based on an idealization—that increments of a particle's position over disjoint time intervals are independent random quantities. This leads to a probabilistic model, the Wiener process, in which a random trajectory, although continuous, is nowhere differentiable. In spite of this physically rather unrealistic feature of the theory, certain formal properties of the nonexistent derivative suggest that the derivative be included in some fashion in mathematical models of physical processes affected by noise. The relevance of Brownian motion is best described heuristically: since the formal derivative of a trajectory would be a limit of Brownian increments, the random values at different times would be stochastically independent. Such a trajectory would represent fluctuations uncorrelated in time, and hence the trajectory would serve as a graph of noise, for example, in electromagnetic transmission problems or electrical system problems. Moreover, the Fourier transform of such a trajectory would be a random function of the frequency with constant variance for all frequencies. That is, the derivative would be a uniform superposition of frequencies and thus would represent "white" noise.

The work of Wiener, Langevin, K. Ito, and others has shown that certain integrals of the derivative may be defined rigorously. Wiener found that definite integrals of white noise weighted by a fixed square-integrable function of the time parameter exist as random functions. Ito extended the definition and developed an elegant theory for integrals where the weight function varies with the trajectory under the restriction that the weight for a fixed time is a function only of the trajectory history up to that time.

The first half of each book under review presents a portion of the Ito theory and the second half includes treatments of two problems associated with mathematical models given by the system:

$$X(t) = \int_0^t A(s)X(s)\,ds + \int_0^t B(s)\,dW(s), \tag{1}$$

$$Y(t) = \int_0^t C(s)X(s)\,ds + \int_0^t D(s)\,dW(s). \tag{2}$$

Here, $W(s)$ denotes a vector whose components are scalar-valued independent Wiener processes and A, B, C, D, are given matrix-valued functions of