BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 81, Number 5, September 1975

SOME MATHEMATICAL PROBLEMS IN BIOLOGY

BY S. I. RUBINOW

ABSTRACT. A survey is presented of some mathematical problems encountered in biological studies. A brief description of the problems to be discussed are as follows. 1. Blood flow: The mean flow of blood through arteries and veins does not behave in accordance with Poiseuille's law all the time. 2. Tracer analysis: The inferences to be drawn from radioactive tracer studies of physiological systems, called compartment analysis, is an inverse problem. 3. Cell populations: The growth of cell populations is conveniently described by differential equations which utilize either age and time or maturity and time as independent variables.

I would like to take this opportunity to tell you about some of the mathematical problems I have encountered in studying biological phenomena during the past ten years or so. These phenomena are all describable by differential equations. However, I think you will see that the problems with which they are associated have novel features when compared to those usually encountered in engineering and the physical sciences. My descriptions will be unsatisfactory in the sense that they will be very brief. However, brevity will permit me to present you with an overview of several biomathematical problems deriving from biological experience.

Steady blood flow. I will describe first a problem in biomechanics, because the latter is one area of biology in which the underlying mathematical theory is very well understood. When the mean steady volume flux of blood through the vena cava of a dog is measured as a function of the pressure difference between the upstream end and the downstream end, a curve is determined [2], as illustrated in Figure 1. Similar observations have been made of viscous fluid flow through a rubber tube [2], as shown in Figure 2. In fact, the observed blood flux through the entire circulatory system of the dog displays similar properties [2]–[5]. In determining the curves of Figures 1 and 2, the upstream pressure was held fixed while the downstream pressure was decreased. The noteworthy feature of these experiments is that when the pressure difference increases beyond a certain value, the flux does not increase, unlike the flow through a rigid tube.

A simple theory of viscous fluid flow through a tube can account for these observations provided the collapsibility of the tube is accounted for [6]. In the

---

This article is the text of an address presented to the American Mathematical Society at its meeting in New York on April 10–13, 1974. It is based on some of the material presented in the author's monograph [1]; received by the editors October 16, 1974.

AMS (MOS) subject classifications (1970). Primary 92A05.

Key words and phrases. Mathematical biology, blood flow, tracer analysis, compartment systems, cell population growth, cell density function.