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ON CUSPIDAL REPRESENTATIONS OF p-ADIC REDUCTIVE GROUPS

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ABSTRACT. Let k be a p-adic field, and G a reductive connected algebraic group over k. Fix a maximal torus T of G which splits in an unramified extension of k, and which has the same split rank as the center of G. For each character θ of $T(k)$, satisfying some conditions, there is a cuspidal representation γ_θ of $G(k)$ which is a sum of a finite number of irreducible representations; the correspondence $\theta \mapsto \gamma_\theta$ is one-to-one on the orbits of such characters by the little Weyl group of T; furthermore, the formulas for the formal degree of γ_θ and its character for sufficiently regular elements of $T(k)$ are given: they are formally the same as is the discrete series for real reductive groups.

1. **Unramified maximal tori.** Let k be a p-adic field, that is a finite extension of $\mathbf{Q}_p$ or a field of formal series over a finite extension of $\mathbf{F}_p$. We denote by $\bar{k}$ the residue field of order q.

Let G be a reductive connected algebraic group defined over k, the derived group G_{der} of which is simply connected. A maximal torus of G defined over k is called *minisotropic* if it normalizes no (proper) horocyclic subgroup of G defined over k.

LEMMA. *Suppose there exists a minisotropic maximal torus T of G which splits in a finite unramified extension L of G. Then the Galois group Γ of L over k has a unique fixed point v in the apartment of T in the building of $G_{\mathrm{der}}(L)$* [2]; *moreover, the face of v is minimal amongst the faces in this apartment which are invariant by Γ.*

2. **Characters.** We conserve notations and hypotheses of §1 and the Lemma. Let θ be a continuous character of $T(k)$. For each $\lambda \in X^{v}(T)$, the lattice of rational one-parameter subgroups of T, we define a character θ_λ of L^x by

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