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GENERIC PROPERTIES OF RETARDED FUNCTIONAL DIFFERENTIAL EQUATIONS[1]

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Consider the retarded functional differential equation (RFDE)

$$\dot{x}(t) = f(x_t) \tag{1}$$

where as in [1], $x_t(\theta) = x(t + \theta), -1 \leqslant \theta \leqslant 0, x_t \in C = C([-1, 0], R^n)$, and $f \in X = C^\infty(C, R^n)$. Oliva [5] showed fixed points of (1) generically are hyperbolic; here we generalize to the theorem of Kupka [2], Markus [3] and Smale [7]. With an appropriate Whitney (Baire) topology on X, we have

THEOREM 1. *The set of $f \in X$ for which*

1. *all fixed points and all periodic solutions of* (1) *are hyperbolic,*
2. *all global unstable manifolds are injectively immersed in C, and*
3. *all global unstable and local stable manifolds intersect transversally*

is a residual subset of X.

The restriction to the local stable manifold in 3. is necessary as we lack backwards existence and uniqueness.

If we consider only equations

$$\begin{gathered} \dot{x}(t) = F(x(t - \tau_1), x(t - \tau_2), \ldots, x(t - \tau_p)), \\ 0 \leqslant \tau_1 < \tau_2 < \cdots < \tau_p \leqslant 1 \text{ fixed}, \end{gathered} \tag{2}$$

with $F \in C^\infty(R^{np}, R^n)$, we obtain

THEOREM 2. *Let $\tau_1 = 0$. Then the set of $F \in C^\infty(R^{np}, R^n)$ for which* 1., 2. *and* 3. *above hold is a residual set.*

The question of what happens when $\tau_1 > 0$ seems to be open.

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