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TOPOLOGICALLY DEFINED CLASSES OF GOING-DOWN DOMAINS

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1. **Introduction.** This note announces some results which build upon the studies of Dobbs [3], [4] and Dobbs and Papick [5] on going-down extensions and going-down domains. Whereas much of [4] was motivated by flatness (cf. [11, 5.D], [15]), the present work has a topological stimulus (cf. [7], [8, Proposition 1.10.13(a), (b′)], [10, pp. 145–160], [12], [14, Corollaire 2, p. 42]). We introduce and study new topologically defined classes of going-down domains, by considering how various going-down conditions on a domain R and its overrings relate to conditions on the topological space $\mathrm{Spec}(R)$.

Details, as well as a systematic study of the behavior of various classes of going-down domains under homomorphic images, localization and globalization, integral change of rings, and the "$D + M$ construction", will appear elsewhere.

2. **Notation.** Let $\mathcal{P}$ (respectively, $\mathcal{Q}$) be a property which may be satisfied by an extension of (commutative integral) domains (respectively, by the map induced on prime spectra by an extension of domains). A domain R is a *$\mathcal{P}$ domain* (*respectively, $\mathcal{Q}$ domain*) if $R \subset T$ (respectively, $\mathrm{Spec}(T) \longrightarrow \mathrm{Spec}(R)$) satisfies $\mathcal{P}$ (respectively, $\mathcal{Q}$) for each overring T of R.

3. **Going-down domains and *i*-domains.** In this section, we introduce tools needed for the remaining sections, and at the same time extend and clarify notions already present in the literature. Recall from [4] and [5] that a domain R is called a *going-down domain* (written R is GD) in case we take $\mathcal{P} =$ GD; and R is said to be *treed* if $\mathrm{Spec}(R)$, as a partially ordered set under inclusion, is a tree. In [4], it is shown that a GD domain must be treed; an example of Lewis, described in [13], shows that the converse need not be true. By taking $\mathcal{P} =$ mated (as defined by Dawson and Dobbs [2]) and $\mathcal{Q} =$ injec-

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