BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 81, Number 4, July 1975

CONVOLUTEURS OF H^p SPACES

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Communicated by Alberto Calderón, March 3, 1975

Nonperiodic analogues and generalizations of some results of Duren and Shields [1] are given. In the process, the key role played by the homogeneous Besov spaces and their images under the Fourier transform will be highlighted. Our results concern the following spaces in addition to the usual L^p space on R^n for $0 < p < \infty$.

Let ϕ be a smooth function (belonging to S, the space of rapidly decreasing functions) such that $\int \phi(x)\,dx = 1$. Set $\phi_t(x) = t^{-n}\phi(xt^{-1})$, and for f defined on R^n, call

$$u(x, t) = \phi_t * f(x), \qquad u^+(x) = \sup_{t>0} |u(x, t)|.$$

A function f defined on R^n belongs to $H^p(R^n)$, $0 < p < \infty$, if and only if $u^+ \in L^p(R^n)$ [2].

For $0 < \alpha < 1$, $f \in \Lambda^\alpha_{a,p}$ iff $(\int(\|\Delta_h f\|_a/|h|^\alpha)^p dh/|h|^n)^{1/p}$ is finite, where $\Delta_h f$ is the difference operator, $\Delta_h f(x) = f(x+h) - f(x)$. The spaces are defined for other values of α by the formula $R^\beta\Lambda^\alpha_{a,p} = \Lambda^{\alpha+\beta}_{a,p}$, where R^β is the Riesz potential of order β defined by closing the operator defined on a subset of S by $(R^\beta f)^\wedge(\xi) = |\xi|^{-\beta}\hat{f}(\xi)$. These spaces are homogeneous Besov spaces [6] in contrast to [9]; for $a = p = 2$, α a nonnegative integer, this is the space of tempered distributions for which all derivatives of order α are in L^2, with no other condition on the lower order derivatives or on the function itself. The characterization that we use most frequently is given in [6]; $f \in \Lambda^\alpha_{p,q}$ iff for k the smallest nonnegative integer greater than $\alpha/2$, if u denotes the temperature with initial value f, and if $M_p(u; t) = \|u(\cdot, t)\|_p$, then

$$\left(\int_0^\infty \left[t^{k-\alpha/2} M_p\left(\frac{\partial^k u}{\partial t^k}; t\right)\right]^q t^{-1}\,dt\right)^{1/q}$$

is finite. For example, $\Lambda^{n(1-1/p)}_{1,1}$ is the containing Banach space B^p of Duren and Shields [1].

Finally, we must consider the Fourier image of these spaces, the spaces

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AMS (MOS) subject classifications (1970). Primary 42A18; Secondary 46J15, 46J35.