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LOOP SPACES AND FINITE ORTHOGONAL GROUPS[1]

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In this note we announce the results of our computations of the mod 2 homology of the orthogonal groups $O(n, \mathbf{F}_q)$ over finite fields $\mathbf{F}_q$ of characteristic $p \neq 2$. We have obtained a 2-local equivalence between the infinite loop space associated with these orthogonal groups and the homotopy fiber $JO(q)$ of the map $(\psi^q - 1)$: $BO(\mathbf{R}) \longrightarrow BSO(\mathbf{R})$, where ψ^q is the Adams operation. For $q \equiv \pm 3 \pmod 8$, these spaces $JO(q)$ are of considerable geometric interest, since $\pi_* JO(q)$ is essentially the image of J_*: $\pi_* SO(\mathbf{R}) \longrightarrow \pi_* SF$ at the prime 2. Here J: $SO(\mathbf{R}) \longrightarrow SF$ is the J-homomorphism of G. Whitehead.

Since the Whitney sum induces an infinite loop space structure on $JO(q)$, we can define Dyer-Lashof operations on its homology. We have computed $H_*(JO(q), \mathbf{Z}_2)$ as an algebra over the Dyer-Lashof algebra.

Our main results are as follows:

THEOREM 1. *There is an equivalence of infinite loop spaces*

$$\Gamma_0 BO(\mathbf{F}_q)_{(2)} \xrightarrow{\simeq} JO(q)_{(2)}.$$

Here $\Gamma_0 BO(\mathbf{F}_q)$ denotes the 0-component of the group completion of $\amalg_{n=0}^{\infty} BO(n, \mathbf{F}_q)$. See May [3] for details.

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