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QUASI-ANALYTIC VECTORS AND QUASI-ANALYTIC FUNCTIONS

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1. **Introduction.** The theory of quasi-analytic classes is a part of function theory that is now over fifty years old. The notion of a quasi-analytic vector is a relatively recent development in operator theory. My purpose here is to discuss the mutual interaction of these ideas, and in particular to show how the operator-theoretic point of view leads in a natural way to broad and interesting generalizations of some of the classical results.

I will begin by recalling some operator theory. Let A be an operator—unbounded in general—with domain $\mathscr{D}(A)$ in a Banach space X. A vector x is a C^∞ *vector* for A if x belongs to $\mathscr{D}^\infty(A)=\bigcap_{n=1}^\infty \mathscr{D}(A^n)$. (Think of the example: $A=D=$differentiation. Then C^∞ vectors are just C^∞ functions.) An *analytic vector* for A is a C^∞ vector x such that the series $\sum_{n=0}^\infty (t^n/n!)\|A^n x\|$ has a positive radius of convergence. This is a growth condition on $\|A^n x\|$; namely, $\|A^n x\|^{1/n}=O(n)$. $\mathscr{D}^a(A)$ will denote the space of analytic vectors for A.

Analytic vectors were introduced by Nelson in 1959 [15]. Among many other things, he proved the following fundamental fact.

1.1. THEOREM A. *Let A be a symmetric operator on a Hilbert space H. If A has a dense set of analytic vectors, then A is essentially selfadjoint (that is, its closure is selfadjoint).*

PROOF. By a well-known theorem of Naĭmark, there is an extension A^0 of A (on a possibly larger Hilbert space $K\supseteq H$) which is selfadjoint. Let $U_t=\exp(itA^0)$ be the one-parameter group generated by A^0.

To show that A is essentially selfadjoint, we must prove that $A+i$ and $A-i$ have dense ranges. Suppose that y is orthogonal to the range of $A+i$. Then in particular y is orthogonal to $(A+i)\mathscr{D}^a(A)$. Then for all $x\in\mathscr{D}^a(A)$, $(Ax, y)=-i(x, y)$. If $x\in\mathscr{D}^a(A)$ then $A^n x\in\mathscr{D}^a(A)$ for all positive integers n, and it follows that

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