BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 81, Number 3, May 1975

EXTENSIONS OF ERGODIC ACTIONS AND GENERALIZED DISCRETE SPECTRUM

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Communicated by P. R. Halmos, February 4, 1975

1. **Introduction.** In this paper we announce results concerning extensions of ergodic actions of locally compact groups. Our results about extensions, together with related results and applications, enable us to obtain a measure theoretic analogue of Furstenberg's work in topological dynamics, to extend several well-known aspects of ergodic theory, and to present a unified view of these various phenomena. The author wishes to thank Professor G. W. Mackey for many helpful suggestions and conversations.

2. **Generalization of the von Neumann-Halmos-Mackey theory.** By an ergodic G-space, where G is a locally compact second countable group, we mean a Lebesgue space (X, μ) together with a Borel action of G on X, under which μ is invariant and ergodic. (X, μ) is called an extension of the ergodic G-space (Y, ν) if there is a Borel G-map $p: X \longrightarrow Y$ with $p_*(\mu) = \nu$. By decomposing μ with respect to ν over the fibers of p, $L^2(X)$ becomes a Borel G-Hilbert bundle over Y [8]. In the study of a single G-space X, an important role is played by the decomposition of the representation of G on $L^2(X)$ defined by translation. For the study of extensions, the decomposition of $L^2(X)$ into G-invariant subbundles over Y plays an analogous role. Theorems A, B, and C are the generalization to extensions of the classical "discrete spectrum" theory of von Neumann and Halmos [4], [9], as generalized, in part, by Mackey [5]. The classical results and Mackey's generalization are included in these theorems as the special case in which Y is one point.

DEFINITION. X has relatively discrete spectrum over Y if $L^2(X)$ is the direct sum of finite dimensional G-invariant subbundles over Y. X has relatively elementary spectrum over Y if each of these subbundles can be taken to be one dimensional.

DEFINITION. If Y is an ergodic G-space and K a compact group, a Borel map $c: Y \times G \longrightarrow K$ is called a homomorphism, or cocycle, if for each g, $h \in G$, $c(y, hg) = c(y, h)c(yh, g)$ for almost all $y \in Y$.

AMS (MOS) subject classifications (1970). Primary 22D40, 28A65, 54H20; Secondary 22D10, 22D30, 47A35.