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WHEN IS A MANIFOLD A LEAF OF SOME FOLIATION?

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Given a connected smooth open manifold L, does there exist a compact manifold M and a C^r codimension q foliation of M with a leaf diffeomorphic to L? Here $1 \leqslant r \leqslant \infty$. Most of our results are for $q = 1$, but note that if the answer is yes for q then it is yes for any $q' > q$. Theorem 1 gives four conditions on L any one of which is sufficient, and the Corollary provides interesting examples where L is a surface. We have found no necessary condition in general, but Theorem 2 gives a strong necessary condition on the ends of L in order that L be a codimension one leaf each of whose ends has only one asymptote. Details and proofs will appear elsewhere.

THEOREM 1. *L is diffeomorphic to a leaf of a C^r codimension q foliation of some compact manifold if any one of the following conditions is satisfied* ($q = 1$ *except possibly in condition* 1.4).

1.1. *L is diffeomorphic to the interior of a compact manifold-with-boundary* ($r = \infty$ *and L will be a proper leaf*).

1.2. *$L = L_1 \# L_2$ where L_1 and L_2 are proper leaves of C^r codimension one foliations of compact orientable manifolds.*

1.3. *$L = L_1 - X$ where L_1 is a leaf of a C^r codimension one foliation of a compact manifold with a closed transversal which intersects L_1 in X.*

1.4. *L is a regular covering space of a compact manifold with covering group which has a C^r action on a connected compact q-manifold with a free orbit.* (*If the orbit is discrete, the leaf L will be proper.*)

Recall (see e.g. [2]) that an *end* ϵ of a connected manifold is determined by a sequence $U_1 \supset U_2 \supset \ldots$ of unbounded components of the complements of compact sets such that $\bigcap_{i=1}^{\infty} \overline{U}_i = \varnothing$. Another such sequence $V_1 \supset V_2 \supset \ldots$ determines the same end if every U_i contains some V_j. Each U_i is called a *neighborhood* of ϵ. Define ϵ to be *boundable* if it has a closed neighborhood of the form $B \times [0, \infty)$ where B is a connected compact manifold.

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