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RICCATI SYSTEMS[1]

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Consider the differential equations

$$y^{(n)} + p(t)y = 0 \tag{1}$$

and

$$y^{(n)} - p(t)y = 0, \tag{2}$$

where p is positive and continuous on $[0, \infty)$, and suppose that, for some $a \geqslant 0$ and some integer $k \in [1, n-1]$ either of these equations has at least one nontrivial solution y for which

$$y(a) = y'(a) = \cdots = y^{(k-1)}(a) = y^{(k)}(c)$$
$$= y^{(k+1)}(c) = \cdots = y^{(n-1)}(c), \qquad c > a.$$

The point $\eta_{k,n-k}(a) = \inf c$, where c ranges over all values for which such solutions exist, is called the "$(k, n-k)$-focal point of a" (the fact that $\eta_{k,n-k}(a) > a$ is elementary). The point $\eta(a) = \min_k \eta_{k,n-k}(a)$ is referred to as "the focal point of a". It is known that equation (1) can only have focal points $\eta_{k,n-k}(a)$ for which $n-k$ is an odd number, while in the case of equation (2) $n-k$ must be even [4]. For the study of focal points we may therefore replace (1) and (2) by the single equation

$$y^{(n)} - (-1)^{n-k} py = 0. \tag{3}$$

In the oscillation theory of equations of the form (1) or (2), focal points play a role very similar to that of the more commonly used conjugate points [1]. In particular, it can be shown that the nonexistence of a focal point $\eta(a) \in (a, \infty)$ is equivalent to the disconjugacy of the equation on $[a, \infty)$.

Our principal result characterizes the focal points of an equation in terms of continuity properties of the solutions of an associated nonlinear differential system.

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