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WEAK CONTINUITY OF BANACH ALGEBRA PRODUCTS

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1. This note is based on the remark that a variety of common Banach algebras are dual Banach spaces. For example, a well-known result of Sakai (see [6]) states that von Neumann algebras are characterized, among the C^*-algebras, by this property (see §3 for further examples).

Clearly, in a dual Banach space two additional topologies are ubiquitous companions to the norm: the *weak** and the bounded *weak** topologies (see [1]). Below we announce several theorems about continuity of the product in these topologies. We thereby obtain known as well as (apparently) new results in concrete instances considered by Pym, Rubel, Shields, Ryff, Shapiro, Conway, Dixmier, et al.

Generally we follow [2] for notations and definitions, with the main exception that we will consider a Banach space A whose dual A^* will be denoted by B; a will be a typical element of A and b and c will be typical elements of B. Also, the bounded weak* topology (see [1]) on B will be denoted by ℓw^* (rather than bw^*) in view of the general setting of [3].

2. Let X, Y and Z be linear topological spaces and let $f: X \times Y \to Z$ be a bilinear map. Consider the following properties of f:

[J, (x_0, y_0)]: $f(x, y) \to f(x_0, y_0)$ as $x \to x_0$, $y \to y_0$;

[J]: f satisfies [J, (x_0, y_0)] for all (x_0, y_0);

[L]: $f(x, y_0) \to f(x_0, y_0)$ as $x \to x_0$ for each y_0;

[R]: $f(x_0, y) \to f(x_0, y_0)$ as $y \to y_0$ for each x_0;

then

$$[\mathrm{J}, (0, 0)] + [\mathrm{L}] + [\mathrm{R}] \Leftrightarrow [\mathrm{J}],$$

and none of the three properties in the left-hand term is redundant in general.

We assume from now on that there is a norm continuous bilinear map

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