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SOME SINGULAR PERTURBATION PROBLEMS

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1. Introduction. The singularly perturbed boundary value problem

$$\epsilon y'' = f(t, y, y', \epsilon), \quad 0 < t < 1, \tag{1.1}$$

$$y(0, \epsilon) = A, \quad y(1, \epsilon) = B, \tag{1.2}$$

for ϵ a small positive parameter, has been studied extensively under various linearity restrictions. See, for example, [3] and [4], and the references therein. However, two principal assumptions have been that the corresponding reduced problem

$$\begin{gathered} 0 = f(t, u, u', 0), \quad 0 < t < 1, \\ u(1) = B, \end{gathered} \tag{1.3}$$

has a solution $u = u(t)$ of class $C^{(2)}[0, 1]$ and that in a suitable tube around u, $f_{y'} = \partial f/\partial y' \leqslant -k$, for some positive constant k. This latter assumption excludes the occurrence of turning points and makes the function u a stable root of (1.3).

Under additional assumptions, by means of several asymptotic methods, the existence of a solution $y = y(t, \epsilon)$ of (1.1), (1.2), for each ϵ sufficiently small, can be deduced and this solution can be shown to satisfy an estimate of the form

$$y(t, \epsilon) = u(t) + O(|A - u(0)| \exp[-kt\epsilon^{-1}]) + O(\epsilon), \quad 0 \leqslant t \leqslant 1.$$

Here O denotes the standard Landau order symbol. The exponential term $v(t, \epsilon) = \exp[-kt\epsilon^{-1}]$ is a boundary layer function, in that $v(0, \epsilon) = 1$ and $v(t, \epsilon) \longrightarrow 0$ as $\epsilon \longrightarrow 0^+$ for $t > 0$.

2. Statement of the problem and main result. Consider the more general boundary value problem

$$a(t, \epsilon) y'' = f(t, y, y', \epsilon), \quad 0 < t < 1, \tag{2.1}$$

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