BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 81, Number 3, May 1975

WHY ANY UNITARY PRINCIPAL SERIES REPRESENTATION OF SL_n OVER A p-ADIC FIELD DECOMPOSES SIMPLY

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Communicated by Hyman Bass, December 2, 1974

Recently A. Knapp [3] announced that every unitary principal series representation of a semisimple Lie group decomposes simply; that is, no two distinct irreducible components of a given unitary principal series representation are equivalent. In proving this result Knapp analyzed in detail the structure of the spaces of intertwining operators for principal series representations. His analysis used a detailed description, due to Harish-Chandra, of the Fourier transform on semisimple Lie groups. In this note we want to prove the analogue of Knapp's result for SL_n over a p-adic field. Our proof will be conceptually quite different from Knapp's. Although the case we consider is admittedly quite special, there is hope that this approach to the problem will generalize.

Let F be a nonarchimedean local field and let $G = GL_n(F)$. Let D be the diagonal subgroup of G and U the upper unipotent matrices. Let $B = D \cdot U$ be the group of all nonsingular upper triangular matrices and write δ for the modular function of B. (If d_lb is a left Haar measure for B, then $\delta(b)d_lb$ is a right Haar measure.) Let K be a maximal compact subgroup of G and recall that $G = KB$.

Let χ be any (unitary) character of D and regard it as a character of B. The induced representation $\pi_\chi = \mathrm{Ind}_B^G(\chi\delta^{1/2})$ is called the (unitary) principal series representation attached to χ. To describe the unitary representation π_χ explicitly we let H_χ denote the Hilbert space of all complex-valued measurable functions h on G such that $h(gb) = \chi^{-1}(b)\delta^{-1/2}(b)h(g)$ $(g \in G,\ b \in B)$ and such that $\int_K |h(k)|^2 dk < \infty$. Then π_χ is just left translation in H_χ: $(\pi_\chi(x)h)(g) = h(x^{-1}g)$ $(h \in H_\chi;\ g, x \in G)$.

We use the subscript "1" to denote the subgroup of $G_1 = SL_n(F)$ ob-

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AMS (MOS) subject classifications (1970). Primary 22E50; Secondary 22E35, 22D10, 22D30.

[1]The authors would like to take this opportunity to thank the SFB Universität Bonn for supporting this research.