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VOLTERRA-STIELTJES INTEGRAL EQUATIONS WITH LINEAR CONSTRAINTS AND DISCONTINUOUS SOLUTIONS

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X and Y denote Banach spaces; we consider systems of the form

$$\text{(K)} \qquad y(t) - y(t_0) + \int_{t_0}^{t} d_\sigma K(t, \sigma) \cdot y(\sigma) = f(t) - f(t_0),$$

$$\text{(F)} \qquad F[y] = c,$$

where $y, f \in G([a, b], X)$ (the space of regulated functions g: $[a, b] \longrightarrow X$, i.e., g has only discontinuities of the first kind); $K \in G^{\mathrm{uo}}$ (see §2) and $F \in L[G([a, b], X), Y]$ (linear constraint). (K) includes linear Volterra integral equations, linear delay differential equations, differential equations $y' + A'y = f^1$, with the meaning that we have

$$\text{(L)} \quad y(t) - y(s) + \int_s^t dA(\sigma) \cdot y(\sigma) = f(t) - f(s) \quad \text{for all } s, t \in [a, b].$$

In §2 we give the existence of the resolvent for (K) and in §3 for (L); in §4 we find the Green function for the system (K), (F). The results of §1 are used in the proofs. All results of this announcement may be extended to open intervals and Y a separated sequentially complete locally convex TVS.

The proofs will appear in [H.3].

1. A *division* of $[a, b]$ is a finite sequence d: $t_0 = a < t_1 < \cdots < t_n = b$. We write $|d| = n$ and $\Delta d = \sup_{1 \leq i \leq n} |t_i - t_{i-1}|$. The set D of all divisions of $[a, b]$ is ordered by refinement and $\lim_{d \in D} x_d$ denotes the limit according to the associated net. For α: $[a, b] \longrightarrow L(X, Y)$ and f: $[a, b] \longrightarrow X$ we define the usual Riemann-Stieltjes operator integral

$$\int_a^b d\alpha(t) \cdot f(t) = \lim_{\Delta d \to 0} \sum_{i=1}^{|d|} [\alpha(t_i) - \alpha(t_{i-1})] \cdot f(\xi_i)$$

where $\xi_i \in [t_{i-1}, t_i]$ (see [G], [H.1], [D]), and the *interior integral*

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