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THE BOOLEAN ALGEBRA OF LOGIC

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A general method of constructing finitely axiomatizable theories is sketched. It is shown that every recursively enumerable Boolean algebra is isomorphic to the Boolean algebra of sentences of some finitely axiomatizable theory. More complete details of the proof will appear in a forthcoming monograph by William Hanf, Dale Myers, and Roger Simons. This verifies Conjecture I of Hanf [2] that every axiomatizable theory is recursively isomorphic to a finitely axiomatizable theory. It solves a problem of [2] by showing that there exists a finitely axiomatizable undecidable theory with countably many complete extensions and shows that Conjecture II is false. Another consequence is that there exists a finitely axiomatizable theory whose Boolean algebra of sentences has an ordered basis of type θ where θ is any constructive ordinal. A complete characterization of $\mathfrak{B}_{\langle 2 \rangle}$, the Boolean algebra of sentences of first order logic with equality and a single binary predicate, is obtained. These last two results answer problems considered by Tarski in the late 1930's and proposed to the author around 1960.

THEOREM 1. *Given any recursively enumerable linear order type α, there exists a finitely axiomatizable theory F whose Boolean algebra of sentences has an ordered basis of type* $1 + \alpha$.

SKETCH OF PROOF. Four two-tape nonwriting Minsky machines (see [4, §1]) are constructed. Each machine has a halt instruction and a nondeterministic branch instruction in addition to the four instructions for incrementing and decrementing the two tapes. Machine A is a universal machine which can interpret the program of any Minsky machine. It will be used to simulate the operation of machines B, C, and D. Each of these machines

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