BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 81, Number 3, May 1975

EXTENSIONS OF THE HASSE NORM THEOREM

BY DENNIS GARBANATI

Communicated by Barbara Osofsky, February 4, 1975

1. **Introduction.** In two works, one in 1930 [5, p. 38] and the other in 1931 [6], H. Hasse produced one of the major theorems of class field theory, namely, his norm theorem which states that if k is a cyclic extension of the number field k' then an element of k' is the norm of an element of k if and only if it is the norm of an element everywhere locally. Also in 1931 [6, p. 68], Hasse showed that for the fields $k' = Q$ (the rationals) and $k = Q(\sqrt{13}, \sqrt{-3})$, his norm theorem did not hold, and hence his theorem, unlike all other major results of class field theory, is not true for arbitrary abelian extensions. In the 1967 publications [1, p. 360] from the Brighton Conferference, J. Tate and J.–P. Serre presented $k = Q(\sqrt{13}, \sqrt{17})$ as another example where the Hasse norm theorem does not hold. In 1971 Y. Furuta produced an equation [4, p. 321] which, were it not for an annoying factor in the denominator, would show when the Hasse norm theorem held for k/Q in terms of the central class number and the genus number of k. See also a very interesting result of O. Taussky-Todd [7, Theorem 5]. It is only natural to ask the following question. For which noncyclic extensions of the rationals Q does the Hasse norm theorem hold? The aim of this note is to present theorems which give a computable answer to this question for a certain class of noncyclic extensions of Q. Detailed proofs of the theorems will appear elsewhere.

2. **A new characterization of the Hasse norm theorem.** *Let k be a finite abelian extension of Q.*

Let p be a prime divisor in Q. Let $\beta \in Q^* = Q\backslash\{0\}$. Let $\left(\frac{\beta, k}{p}\right)$ be the Hasse norm residue symbol. Let N be the norm map from k to Q. By abuse of language we will take the statement "the Hasse norm theorem holds for k" to mean that for each $\beta \in Q^*$ there exists $\hat{\beta} \in k$ such that $N\hat{\beta} = \beta$ if and only if $\left(\frac{\beta, k}{p}\right) = 1$ for all prime divisors p of Q.

Let K' be the "narrow" genus field of k, i.e. the maximal abelian ex-

---

AMS (MOS) subject classifications (1970). Primary 12A65; Secondary 12A50.