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AN ATTACK ON RIGIDITY. I, II

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1. **Introduction.** We are primarily interested in the continuous rigidity (as opposed to the infinitesimal rigidity) of polyhedral surfaces in three-space. In 1813 Cauchy proved (with a few patchable mistakes) that any two convex polyhedral surfaces that are isometric are congruent, and even today his result has been improved only infinitesimally. If one looks at strictly convex embeddings this implies that they are rigid (see Gluck [5] for definitions). However, since Cauchy, most efforts have been devoted to either infinitesimal rigidity (see Dehn [4] for instance) or uniqueness of embeddings in a class closely resembling convex embeddings (cf. Stoker [9], Alexandrov [1], or Pogorelov [8]) even in the smooth category, where more "modern" methods have supplanted Cauchy's (see Nirenberg [7], Chern [3], or Herglotz [6] for instance). These techniques seem not too promising for the old conjecture that all embedded (or immersed) polyhedral (or smooth) surfaces are continuously rigid.

We present some ideas and techniques which we hope will be useful for the more general rigidity problem. Among other things, we show that any embedded suspension of a polygonal circle is rigid, as well as Theorems 1 and 2 below.

2. **The structural equations.** We regard a polyhedron P as a finite collection of points $p_1, p_2, \cdots$ in $\mathbf{R}^3$, together with certain unordered pairs of the points, which we call edges. If P corresponds to a triangulated 2-dimensional surface, then the subpolyhedron of points adjacent to a point p corresponds to a circle. Let $p_1, p_2, \cdots, p_n$ denote those points as one proceeds cyclicly around the "link" of p. Let $e_j = p_j - p, j = 1, \cdots, n$, be thought of as a vector. Choose some convenient reference vector, R, and let π: $\mathbf{R}^3 \to R^{\perp}$ denote orthogonal projection onto the plane perpendicular to R. Let θ_{jj+1} denote the angle from $\pi(e_j)$ to $\pi(e_{j+1})$. The idea is to write θ_{jj+1}, or more conveniently $e^{i\theta_{jj+1}}$, as some reasonable function of $z_j = R \cdot e_j, z_{j+1} = R \cdot e_{j+1}$,

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