SOME DEGENERATIONS OF A COMPACT
RIEMANN SURFACE OF GENUS 4

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ABSTRACT. A number of degenerations of a compact hyperelliptic
Riemann surface of genus 4 are studied, using theta function techniques.

1. Let (S, Γ, Δ) be a hyperelliptic Riemann surface of genus 4 with a canonical homology basis. Then S has a representation as a two-sheeted cover of the sphere with ten branch points. We can arrange for the surface to have branch points over $0, 1, \infty, 1/\lambda_1, 1/\lambda_2, \cdots, 1/\lambda_7$, where its real branch points other than 0 and 1 (if any) are all greater than 1 and in ascending order (see [2]). We obtain a concrete representation of (S, Γ, Δ) which we henceforth assume is that illustrated in Figure 1.
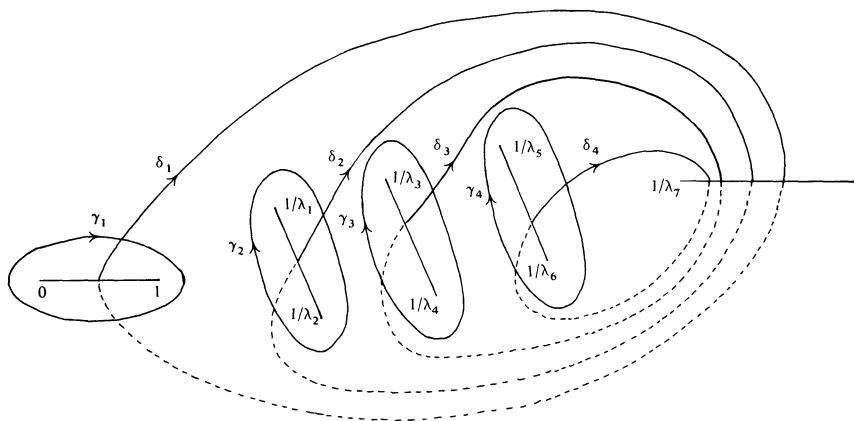

FIGURE 1

For all introductory material we refer the reader to [1], [2] and [3].

There are, for (S, Γ, Δ), precisely ten even theta functions which vanish at the origin (see [1]). We will insist in our degeneration process that these even theta functions remain zero at 0, thus insuring that the surface remains hyperelliptic.