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CLASSIFYING RELATIVE EQUILIBRIA. II

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We announce several theorems which suggest a minimal classification of relative equilibria in the planar n-body problem. These theorems also answer several questions on the nature of degenerate relative equilibria classes which were asked recently by S. Smale [3]. A summary of previous results can be found in an earlier paper [1]. It is a pleasure to thank S. Smale for encouragement in this work.

1. **Morse theory and relative equilibria.** We study the critical set of a real analytic function $\tilde{V}_m < 0$ on a real analytic manifold X_m where $n \geqslant 3$ and $m = (m_1, \ldots, m_n) \in R^n_+$ are fixed. Critical points of $\tilde{V}_m$ correspond in a 1-1 fashion to classes of relative equilibria. $\tilde{V}_m$ always has a compact critical set which we may investigate by Morse theory even when degenerate critical points exist [2].

The integral singular homology of X_m (a manifold which is homeomorphic to a Stein manifold $P_{n-2}(C) - \tilde{\Delta}_{n-2}$) is given by a recurrence relation [1]. This suggests that there is a uniform lower bound on the number of critical points of each index of $\tilde{V}_m$ which is given by recurrence. As a first step toward classifying relative equilibria Theorem 1 gives such a relation.

In Theorem 2 we assert that $\tilde{V}_m$ is a Morse function for any $n \geqslant 3$ and for almost all $m \in R^n_+$ (in the sense of Lebesgue measure).

Theorem 3 answers the question: Is $\tilde{V}_m$ always a Morse function?

Finally, we examine the case of four masses to show how a degeneracy of $\tilde{V}_m$ arises. An interpretation of Theorem 1 in the degenerate case sheds light on the creation and annihilation of relative equilibria.

2. **Main theorems.** In this paragraph for any i, $0 \leqslant i \leqslant 2n - 4$, let $\mu_i(n)$ denote a uniform lower bound to the number of critical points of $\tilde{V}_m$

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