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MATRIX DIFFERENTIAL EQUATIONS

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Let Ω be the set of n by n matrices with complex elements, let R denote the set of reals, and let R_0 denote the interval $[0, t_0)$ for some $t_0 > 0$. We consider the differential relation

$$(1) \qquad 0 \in z' - f(t, z), \qquad t \in R_0$$

where $z(t) \in \Omega$ and f is a function from $R_0 \times \Omega$ to subsets of Ω. The equation can be interpreted in two senses: Either z is absolutely continuous and the relation holds almost everywhere, or z is continuous and the relation holds except in a countable set.

A function $\phi(t, \rho)$ from $R_0 \times R$ to R is a uniqueness function if the upper solution of the equation

$$(2) \qquad D^+ \rho = \phi(t, \rho), \qquad t \in R_0; \quad \rho(0) = 0$$

is $\rho = 0$. Here D^+ denotes the upper right Dini derivate, though other derivates could be used just as well. The equation (2) is interpreted in the same sense as (1).

We use $|\xi|$ for the Euclidean length of the complex vector ξ, so that $|\xi|^2 = \xi^*\xi$. For $z \in \Omega$ a norm and Kamke norm are defined respectively by

$$\|z\| = \sup|z\xi|, \qquad [z] = \sup \operatorname{Re}(\xi^* z \xi), \quad (|\xi| = 1).$$

We say that f satisfies a uniqueness condition if there exist an $\epsilon > 0$ and a uniqueness function ϕ such that

$$x \in f(t, u), \qquad y \in f(t, v), \qquad \|u - v\| < \epsilon$$

together imply

$$[(u - v)^*(x - y)] \leqslant \|u - v\| \phi(t, \|u - v\|).$$

The hypotheses and conclusions of our theorems hold for $t \in R_0$ and, for simplicity, all coefficients in the examples are integrable.

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