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THE DETERMINANT OF A RANDOM MATRIX

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An experimental worker measures $n \times n$ independent magnitudes a_{ij} $(i, j = 1, \cdots, n)$ and computes the determinant of the corresponding matrix before discarding the a_{ij}'s. One can imagine a set-up where he only gets to know the determinant but not the a_{ij} themselves. By repeating the same experiment over and over and averaging the corresponding determinants he obtains the determinant of the unknown mean values m_{ij} of the magnitudes a_{ij}.

The purpose of this note is to indicate that under reasonable assumptions he can get much more information about the *unknown matrix* m_{ij} *at no extra cost.*

Assume that the entries a_{ij} $(i, j = 1, \cdots, n)$ are jointly Gaussian variables with *unknown* means m_{ij} and *known* correlation matrix R.

For the remainder of this note we concentrate on the special case where the entries are independent and have the same variance, i.e. $R = \rho I$ $(\rho \neq 0)$; this case already well illustrates our point. The results given here illustrate the fact that in many nonlinear identification problems the presence of noise can prove helpful (see [1]). We give first a complete analysis of the 2×2 case.

For the discussion below put

$$M(x) = \begin{pmatrix} a + x_1 & b + x_2 \\ c + x_3 & d + x_4 \end{pmatrix}$$

with a, b, c, d *unknown* constants, and x_1, x_2, x_3, x_4 Gaussian (0, 1) random variables. The case $(0, \rho)$ $(\rho \neq 0)$, is just the same.

Bring in the characteristic function of the determinant $F(\lambda) = Ee^{i\lambda \det M}$ and conclude after some elementary computation that

$$F(\lambda) = e^{i\lambda M(0)}\Psi(a + d, -\lambda)\Psi(d - a, \lambda)\Psi(b + c, -\lambda)\Psi(b - c, \lambda).$$

Here

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