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LINEAR APPROXIMATION BY EXPONENTIAL SUMS ON FINITE INTERVALS

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Let $\Lambda = \{\lambda_k\}_{k=1}^{\infty}$ be a sequence of distinct nonnegative real numbers. It is well known that the exponential sums

$$e_s(x) = \sum_{k=1}^{s} a_k e^{\lambda_k t}, \quad a_k \in R,\ s = 1, 2, \cdots, \tag{1}$$

are dense in $C[A, B]$, $-\infty < A < B < +\infty$, if and only if Müntz' condition $\Sigma_{\lambda_k \neq 0} 1/\lambda_k = +\infty$ holds. In this note Jackson-type results on the rate of convergence of the exponential sums (1) are given. Substituting

$$x = e^{t-B}, \quad t \in [A, B],\ x \in [a, 1], \tag{2}$$

where $a = e^{A-B}$, we are led to the problem where the functions $f \in C[a, 1]$, $0 < a < 1$, are to be approximated on $[a, 1]$ by the Λ-polynomials

$$p_s(x) = \sum_{k=1}^{s} b_k x^{\lambda_k}, \quad b_k \in R,\ s = 1, 2, \cdots. \tag{3}$$

Recently, many optimal or almost optimal Jackson-Müntz theorems on the approximation properties of the Λ-polynomials (3) for the interval $[0, 1]$ have been published (cf. J. Bak and D. J. Newman [1] and M. v. Golitschek [2]). Considering intervals $[a, 1]$, $a > 0$, one would expect that the Λ-polynomials have even better approximation properties than on $[0, 1]$, as the "singular" point $x = 0$ might have less influence. Theorems 1 and 2 prove this conjecture.

THEOREM 1. *Let $0 < a < 1, M > 0$. If Λ satisfies*

$$0 \leqslant \lambda_k \leqslant Mk \quad \textit{for all } k = 1, 2, \cdots, \tag{4}$$

then for each function $f \in C^r[a, 1]$, $r \geqslant 0$, and each integer $s \geqslant r + 1$ there exists a Λ-polynomial p_s such that for all $a \leqslant x \leqslant 1$

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