BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 81, Number 2, March 1975

COMPLETE AUTOMORPHISM GROUPS

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Communicated by Hyman Bass, October 31, 1974

Let $A(G)$ denote the automorphism group of the group G. If the center of G is trivial, the action of G on itself by conjugation identifies G with the group of inner automorphisms, which is a normal subgroup of $A(G)$. Since $A(G)$ is again centerless, this gives rise to an automorphism tower

$$G \lhd A(G) \lhd A(A(G)) = A^2(G) \lhd \cdots$$

which may continue transfinitely. A group is termed *complete* if it is centerless and if every automorphism is inner. Thus the automorphism tower of a centerless group G is finite if and only if some $A^r(G)$ is complete. Burnside [2, p. 95] had observed that $A(G)$ is complete if and only if G is a characteristic subgroup; that is, if and only if G is normal in $A^2(G)$. He also established [2, p. 96] that a centerless simple group has a complete automorphism group. It was shown by Wielandt [14] that every finite group with trivial center has a finite automorphism tower. In contrast to these results, Hulse [8] proved that the automorphism tower of a centerless polycyclic—and therefore linear—group need not terminate after finitely many steps, although the tower is in this case at most countable.

The work to be described here was motivated by Gilbert Baumslag's conjecture that a sufficiently symmetric group was likely to have a very short tower, and specifically that this would be the case for a free group. We established

THEOREM 1 [3]. *$A(F)$ is complete if F is a free group of finite rank $n \geqslant 2$.*

This confirms Baumslag's conjecture in the sharpest sense: the automorphism tower of a free group stabilizes in precisely one step. Theorem 1 is rather special, in that we had available considerable information about $A(F)$,

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AMS (MOS) subject classifications (1970). Primary 20F55, 20E25; Secondary 20E05, 20E15, 20F15.

[1]The second author was supported by NSF Grant GP-36418X1.