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A RADIAL MULTIPLIER AND A RELATED KAKEYA MAXIMAL FUNCTION

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In this paper we state some results for a maximal function and a Fourier multiplier that are connected with the Bochner-Riesz spherical summation of multiple Fourier series (see Fefferman [3], [5]). Our purpose will be to get sharp estimates for the norm of these operators in dimension two. Proofs will appear elsewhere [2].

Let $N \geqslant 1$ be a real number. By a rectangle of eccentricity N we mean a rectangle R such that

$$\frac{\text{Length of the bigger side of } R}{\text{Length of the smaller side of } R} = N.$$

We will define the direction of R as the direction of its bigger side.

Given a locally integrable function f we consider the maximal function

$$Mf(x) = \operatorname*{Sup}_{x \in R} \frac{1}{|R|} \int_R |f(y)|\, dy,$$

where the "Sup" is taken over rectangles of eccentricity N, but arbitrary direction.

THEOREM 1. *The sublinear operator M is bounded in $L^2(R^2)$ and there exists a constant C, independent of N, such that*

$$\|Mf\|_2 \leqslant C(\log 3N)^2 \|f\|_2.$$

Suppose that m_0 is a smooth function on R with support on $(-1, 1)$ and let $m(r) = m_0(\delta^{-1}(r-1))$, where $\delta > 0$ is a small number.

Consider the Fourier multiplier defined by

$$\widehat{Tf}(\xi) = m(|\xi|)\hat{f}(\xi), \quad f \in C_0^\infty(R^2).$$

THEOREM 2. *There exists a constant C, independent of δ, such that*

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