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MANIFOLDS WITH PREASSIGNED CURVATURE— A SURVEY

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In this paper I discuss two problems of Riemannian geometry in the large concerning the existence of manifolds with preassigned curvature.

The Minkowski problem and its generalization asks in Euclidean space for a closed convex hypersurface whose curvature has been given in advance. The converse to the Gauss-Bonnet theorem asks for the existence, on a two-dimensional manifold, of a Riemannian metric with prescribed Gaussian curvature. The questions have a meeting point: the search for two-spheres in three-space with given strictly positive curvature.

While the first problem goes back to the work of Minkowski [32] in 1897, the second is of more recent vintage: it was posed explicitly by Warner in the early 1960's. Both have been solved in the last few years, and in this survey I try to give an overview and some of the details.

The paper is organized into the following sections:

1. The Minkowski problem
2. The generalized Minkowski problem
3. Converse to the Gauss-Bonnet theorem for smooth manifolds
4. Converse to the Gauss-Bonnet theorem for PL manifolds
5. Realization in three-space

1. The Minkowski problem.

(1.1) CURVATURE OF CONVEX HYPERSURFACES. Let M^n be a smooth closed convex hypersurface in Euclidean space R^{n+1}. The Gauss map $\gamma: M^n \to S^n$ associates with each point $x \in M$ the unit outward normal vector to M at x. Given a region A on M, the ratio

$$\frac{\text{area of } \gamma(A) \text{ on } S^n}{\text{area of } A \text{ on } M^n}$$

represents the average curvature of M throughout the region A, and its

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