BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 81, Number 1, January 1975

MAXIMUM PRINCIPLES WITHOUT DIFFERENTIABILITY[1]

BY FRANK H. CLARKE

Communicated by Alistair Lachlan, August 1, 1974

Introduction. For reasons both theoretical and practical, there has arisen an interest in variational problems not possessing the customary differentiability hypotheses. The author developed in his dissertation a theory of necessary conditions for certain general problems in optimal control and the calculus of variations. In this article we describe the kind of results that may be obtained when this theory is applied to control problems lacking differentiability or smoothness. In §1 we describe the problem and some terminology, and state a theorem similar to the Pontryagin Maximum Principle. §2 gives a "deparametrized" form of the necessary conditions. In neither case are the results stated in their greatest possible generality. Proofs, details and more general results will appear elsewhere.

1. Let there be given functions l: $R^n \longrightarrow R$, g: $[0, 1] \times R^n \times R^m \longrightarrow R$ and f: $[0, 1] \times R^n \times R^m \longrightarrow R^n$, as well as a *multifunction* U: $[0, 1] \longrightarrow R^m$ (i.e. a mapping from $[0, 1]$ to the subsets of R^m).

An admissible *control-response pair* is a pair (u, x) such that u is a (Lebesgue) measurable function from $[0, 1]$ to R^m with $u(t) \in U(t)$ a.e., x is an absolutely continuous function from $[0, 1]$ to R^n, and

$$\dot{x}(t) = f(t, x(t), u(t)) \quad \text{a.e.}$$

The illustrative optimal control problem we consider is

(1) minimize $l(x(1)) + \int_0^1 g(t, x(t), u(t))dt$ over the control-response pairs (u, x) satisfying $x(0) \in S$, where S is a given closed subset of R^n.

DEFINITION 1. Let ϕ: $R^n \longrightarrow R$ be Lipschitz continuous. The *generalized gradient* of ϕ at s, denoted $\partial\phi(s)$, is the set

---

AMS (MOS) subject classifications (1970). Primary 49B10, 49B35.

Key words and phrases. Optimal control, maximum principle, nondifferentiable functions.

[1] Part of this paper is taken from the author's Ph. D. thesis, written under the supervision of R. T. Rockafellar, University of Washington, June 1973.